

# Pari-GP reference card

(PARI-GP version 2.17.1)

Note: optional arguments are surrounded by braces {}.  
To start the calculator, type its name in the terminal: **gp**  
To exit **gp**, type **quit**, **\q**, or **<C-D>** at prompt.

## Help

describe function *?function*  
extended description *??keyword*  
list of relevant help topics *???pattern*  
name of GP-1.39 function *f* in GP-2.\* *whatnow(f)*

## Input/Output

previous result, the result before *%, %~, %`*, etc.  
*n*-th result since startup *%n*  
separate multiple statements on line *;*  
extend statement on additional lines *\*  
extend statements on several lines *{seq1; seq2;}*  
comment */\* ... \*/*  
one-line comment, rest of line ignored *\\ ...*

## Metacommands & Defaults

set default *d* to *val* *default({d},{val})*  
toggle timer on/off *#*  
print time for last result *##*  
print defaults *\d*  
set debug level to *n* *\g n*  
set memory debug level to *n* *\gm n*  
set *n* significant digits / bits *\p n, \pb n*  
set *n* terms in series *\ps n*  
quit GP *\q*  
print the list of PARI types *\t*  
print the list of user-defined functions *\u*  
read file into GP *\r filename*  
set debuglevel for domain *D* to *n* *setdebug(D,n)*

## Debugger / break loop

get out of break loop *break* or **<C-D>**  
go up/down *n* frames *dbg\_up({n}), dbg\_down*  
set break point *breakpoint()*  
examine object *o* *dbg\_x(o)*  
current error data *dbg\_err()*  
number of objects on heap and their size *getheap()*  
total size of objects on PARI stack *getstack()*

## PARI Types & Input Formats

**t\_INT**. Integers; hex, binary *±31; ±0x1F, ±0b101*  
**t\_REAL**. Reals *±3.14, 6.022 E23*  
**t\_INTMOD**. Integers modulo *m* *Mod(n,m)*  
**t\_FRAC**. Rational Numbers *n/m*  
**t\_FFELT**. Elt in finite field **F<sub>q</sub>** *ffgen(q,'t)*  
**t\_COMPLEX**. Complex Numbers *x + y\*I*  
**t\_PADIC**. *p*-adic Numbers *x + 0(p^k)*  
**t\_QUAD**. Quadratic Numbers *x + y\*quadgen(D,{'w})*  
**t\_POLMOD**. Polynomials modulo *g* *Mod(f,g)*  
**t\_POL**. Polynomials *a \* x^n + ... + b*  
**t\_SER**. Power Series *f + 0(x^k)*  
**t\_RFRAC**. Rational Functions *f/g*  
**t\_QFB**. Binary quadratic form *Qfb(a,b,c)*  
**t\_VEC/t\_COL**. Row/Column Vectors *[x,y,z], [x,y,z]~*  
**t\_VEC** integer range *[1..10]*

**t\_VECSMALL**. Vector of small ints *Vecsmall([x,y,z])*  
**t\_MAT**. Matrices *[a,b;c,d]*  
**t\_LIST**. Lists *List([x,y,z])*  
**t\_STR**. Strings *"abc"*  
**t\_INFINITY**.  $±∞$  *+oo, -oo*

## Reserved Variable Names

$π ≈ 3.14, γ ≈ 0.57, C ≈ 0.91, I = √{-1}$  **Pi, Euler, Catalan, I**  
Landau's big-oh notation **O**

## Information about an Object, Precision

PARI type of object *x* *type(x)*  
length of *x* / size of *x* in memory *#x, sizebyte(x)*  
real precision / bit precision of *x* *precision(x), bitprecision(x)*  
*p*-adic, series prec. of *x* *padicprec(x,p), serprec(x,v)*  
current dynamic precision *getlocalprec, getlocalbitprec*

## Operators

basic operations *+, -, \*, /, ^, sqr*  
*i←i+1, i←i-1, i←i\*j, ...* *i++, i--, i\*=j, ...*  
Euclidean quotient, remainder *x\y, x%y, x%y, divrem(x,y)*  
shift *x* left or right *n* bits *x<<n, x>>n* or *shift(x,±n)*  
multiply by  $2^n$  *shiftmul(x,n)*  
comparison operators *<=, <, >=, >, ==, !=, ==, lex, cmp*  
boolean operators (or, and, not) *||, &&, !*  
bit operations *bitand, bitneg, bitor, bitxor, bitneg*  
maximum/minimum of *x* and *y* *max(x,y), min(x,y)*  
sign of *x* (gives  $-1, 0, 1$ ) *sign(x)*  
binary exponent of *x* *exponent(x)*  
derivative of *f*, 2nd derivative, etc. *f', f'', ...*  
differential operator *diffop(f,v,d,{n=1})*  
quote operator (formal variable) *'x*  
assignment *x = value*  
simultaneous assignment  $x ← v[1], y ← v[2]$  *[x,y] = v*

## Select Components

*Caveat*: components start at index  $n = 1$ .  
*n*-th component of *x* *component(x,n)*  
*n*-th component of vector/list *x* *x[n]*  
components *a, a + 1, ... , b* of vector *x* *x[a..b]*  
(*m, n*)-th component of matrix *x* *x[m,n]*  
row *m* or column *n* of matrix *x* *x[m,], x[,n]*  
numerator/denominator of *x* *numerator(x), denominator(x)*

## Random Numbers

random integer/prime in  $[0, N[$  *random(N), randomprime(N)*  
get/set random seed *getrand, setrand(s)*

## Conversions

to vector, matrix, vec. of small ints **Col/Vec, Mat, Vecsmall**  
to list, set, map, string **List, Set, Map, Str**  
create ( $x \bmod y$ ) **Mod(x,y)**  
make *x* a polynomial of *v* **Pol(x, {v})**  
variants of **Pol** *et al.*, in reverse order **Polrev, Vecrev, Colrev**  
make *x* a power series of *v* **Ser(x, {v})**  
convert *x* to simplest possible type **simplify(x)**  
object *x* with real precision *n* **precision(x,n)**  
object *x* with bit precision *n* **bitprecision(x,n)**  
set precision to *p* digits in dynamic scope **localprec(p)**  
set precision to *p* bits in dynamic scope **localbitprec(p)**

## Character strings

convert to TeX representation **strtex(x)**  
string from bytes / from format+args **strchr, sprintf**  
split string / join strings **strsplit, strjoin**  
convert time *t* ms. to h, m, s, ms format **strtime(t)**

## Conjugates and Lifts

conjugate of a number *x* **conj(x)**  
norm of *x*, product with conjugate **norm(x)**  
 $L^p$  norm of *x* ( $L^∞$  if no *p*) **normlp(x, {p})**  
square of  $L^2$  norm of *x* **norml2(x)**  
lift of *x* from Mods and *p*-adics **lift, centerlift(x)**  
recursive lift **liftall**  
lift all **t\_INT** and **t\_PADIC** ( $→t\_INT$ ) **liftint**  
lift all **t\_POLMOD** ( $→t\_POL$ ) **lifttpol**

## Lists, Sets & Maps

**Sets** (= row vector with strictly increasing entries w.r.t. **cmp**)  
intersection of sets *x* and *y* **setintersect(x,y)**  
set of elements in *x* not belonging to *y* **setminus(x,y)**  
symmetric difference  $xΔy$  **setdelta(x,y)**  
union of sets *x* and *y* **setunion(x,y)**  
does *y* belong to the set *x* **setsearch(x,y, {flag})**  
set of all  $f(x,y), x ∈ X, y ∈ Y$  **setbinop(f,X,Y)**  
is *x* a set? **setisset(x)**

**Lists**. create empty list:  $L = \text{List}()$

append *x* to list *L* **listput(L,x, {i})**  
remove *i*-th component from list *L* **listpop(L, {i})**  
insert *x* in list *L* at position *i* **listinsert(L,x,i)**  
sort the list *L* in place **listsort(L, {flag})**

**Maps**. create empty dictionary:  $M = \text{Map}()$

attach value *v* to key *k* **mapput(M,k,v)**  
recover value attach to key *k* or error **mapget(M,k)**  
is key *k* in the dict? (set *v* to  $M(k)$ ) **mapisdefined(M,k, {&v})**  
evaluate *f* at  $M(k)$  **mapapply(M,k,f)**  
remove *k* from map domain **mapdelete(M,k)**

## GP Programming

### User functions and closures

*x, y* are formal parameters; *y* defaults to **Pi** if parameter omitted;  
*z, t* are local variables (lexical scope), *z* initialized to 1.

**fun(x, y=Pi) = my(z=1, t); seq**  
**fun = (x, y=Pi) -> my(z=1, t); seq**

attach help message *h* to *s* **addhelp(s,h)**  
undefine symbol *s* (also kills help) **kill(s)**

**Control Statements** (*X*: formal parameter in expression *seq*)  
if  $a ≠ 0$ , evaluate *seq<sub>1</sub>*, else *seq<sub>2</sub>* **if(a, {seq1}, {seq2})**

eval. *seq* for  $a ≤ X ≤ b$  **for(X = a, b, seq)**  
... for  $X ∈ v$  **foreach(v, X, seq)**  
... for primes  $a ≤ X ≤ b$  **forprime(X = a, b, seq)**  
... for primes  $≡ a \pmod q$  **forprimestep(X = a, b, q, seq)**  
... for composites  $a ≤ X ≤ b$  **forcomposite(X = a, b, seq)**  
... for  $a ≤ X ≤ b$  stepping *s* **forstep(X = a, b, s, seq)**  
... for *X* dividing *n* **fordiv(n, X, seq)**  
...  $X = [n, factor(n)], a ≤ n ≤ b$  **forfactored(X = a, b, seq)**  
... as above, *n* squarefree **forsquarefree(X = a, b, seq)**  
...  $X = [d, factor(d)], d | n$  **fordivfactored(n, X, seq)**  
multivariable **for**, lex ordering **forvec(X = v, seq)**

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loop over partitions of  $n$   
... permutations of  $S$   
... subsets of  $\{1, \dots, n\}$   
...  $k$ -subsets of  $\{1, \dots, n\}$   
... vectors  $v, q(v) \leq B; q > 0$   
...  $H < G$  finite abelian group  
evaluate  $seq$  until  $a \neq 0$   
while  $a \neq 0$ , evaluate  $seq$   
exit  $n$  innermost enclosing loops  
start new iteration of  $n$ -th enclosing loop  
return  $x$  from current subroutine

## Exceptions, warnings

raise an exception / warning  
type of error message  $E$   
try  $seq_1$ , evaluate  $seq_2$  on error

## Functions with closure arguments / results

number of arguments of  $f$   
select from  $v$  according to  $f$   
apply  $f$  to all entries in  $v$   
evaluate  $f(a_1, \dots, a_n)$   
evaluate  $f(\dots f(f(a_1, a_2), a_3) \dots, a_n)$   
calling function as closure

## Sums & Products

sum  $X = a$  to  $X = b$ , initialized at  $x$   
sum entries of vector  $v$   
product of all vector entries  
sum  $expr$  over divisors of  $n$   
... assuming  $expr$  multiplicative  
product  $a \leq X \leq b$ , initialized at  $x$   
product over primes  $a \leq X \leq b$

## Sorting

sort  $x$  by  $k$ -th component  
min.  $m$  of  $x$  ( $m = x[i]$ ), max.  
does  $y$  belong to  $x$ , sorted wrt.  $f$   
 $\prod g^x \rightarrow$  factorization ( $\Rightarrow$  sorted, unique  $g$ )

## Input/Output

print with/without  $\backslash n$ , TeX format  
pretty print matrix  
print fields with separator  
formatted printing  
write  $args$  to file  
write  $x$  in binary format  
read file into GP  
... return as vector of lines  
... return as vector of strings  
read a string from keyboard

## Files and file descriptors

File descriptors allow efficient small consecutive reads or writes from or to a given file. The argument  $n$  below is always a descriptor, attached to a file in **r**(ead), **w**(rite) or **a**(ppend) mode.  
get descriptor  $n$  for file  $path$  in given  $mode$   
... from shell  $cmd$  output (pipe)

close descriptor  
commit pending write operations  
read logical line from file  
... raw line from file  
write  $s \backslash n$  to file  
... write  $s$  to file

forpart( $p = n, seq$ )  
forperm( $S, p, seq$ )  
forsubset( $n, p, seq$ )  
forsubset( $[n, k], p, seq$ )  
forqfvec( $v, q, b, seq$ )  
forsubgroup( $H = G$ )  
until( $a, seq$ )  
while( $a, seq$ )  
break( $\{n\}$ )  
next( $\{n\}$ )  
return( $\{x\}$ )

error(), warning()  
errname( $E$ )  
iferr( $seq_1, E, seq_2$ )

arity( $f$ )  
select( $f, v$ )  
apply( $f, v$ )  
call( $f, a$ )  
fold( $f, a$ )  
self()

sum( $X = a, b, expr, \{x\}$ )  
vecsum( $v$ )  
vecprod( $v$ )  
sumdiv( $n, X, expr$ )  
sumdivmult( $n, X, expr$ )  
prod( $X = a, b, expr, \{x\}$ )  
prodeuler( $X = a, b, expr$ )

vecsort( $x, \{k\}, \{fl = 0\}$ )  
vecmin( $x, \{\&i\}$ ), vecmax  
vecsearch( $x, y, \{f\}$ )  
matreduce( $m$ )

print, print1, printtex  
printp  
printsep( $sep, \dots$ ), printsep1  
printf()  
write, write1, writetex( $file, args$ )  
writebin( $file, x$ )  
read( $\{file\}$ )  
readvec( $\{file\}$ )  
readstr( $\{file\}$ )  
input()

fileopen( $path, mode$ )  
fileextern( $cmd$ )  
fileclose( $n$ )  
fileflush( $n$ )  
fileread( $n$ )  
filereadstr( $n$ )  
filewrite( $n, s$ )  
filewrite1( $n, s$ )

## Timers

CPU time in  $ms$  and reset timer  
CPU time in  $ms$  since gp startup  
time in  $ms$  since UNIX Epoch  
timeout command after  $s$  seconds

## Interface with system

allocates a new stack of  $s$  bytes  
alias  $old$  to  $new$   
install function from library  
execute system command  $a$   
... and feed result to GP  
... returning GP string  
get \$VAR from environment  
expand env. variable in string

## Parallel evaluation

These functions evaluate their arguments in parallel (pthreads or MPI); args. must not access global variables (use **export** for this) and must be free of side effects. Enabled if threading engine is not *single* in gp header.

evaluate  $f$  on  $x[1], \dots, x[n]$   
evaluate closures  $f[1], \dots, f[n]$   
as **select**  
as **sum**  
as **vector**  
eval  $f$  for  $i = a, \dots, b$   
... for each element  $x$  in  $v$   
... for  $p$  prime in  $[a, b]$   
... for  $p = a \bmod q$   
... for  $i = a, a + s, \dots, b$   
... multivariate  
export  $x$  to parallel world  
... all dynamic variables  
frees exported value  $x$   
... all exported values

## Linear Algebra

dimensions of matrix  $x$   
multiply two matrices  
... assuming result is diagonal  
concatenation of  $x$  and  $y$   
extract components of  $x$   
transpose of vector or matrix  $x$   
adjoint of the matrix  $x$   
eigenvectors/values of matrix  $x$   
characteristic/minimal polynomial of  $x$   
trace/determinant of matrix  $x$   
permanent of matrix  $x$   
Frobenius form of  $x$   
QR decomposition  
apply **matqr**'s transform to  $v$

## Constructors & Special Matrices

$\{g(x) : x \in v \text{ s.t. } f(x)\}$   
 $\{x : x \in v \text{ s.t. } f(x)\}$   
 $\{g(x) : x \in v\}$   
row vec. of  $expr$  eval'ed at  $1 \leq i \leq n$   
col. vec. of  $expr$  eval'ed at  $1 \leq i \leq n$   
vector of small ints

gettime()  
getabstime()  
getwalltime()  
alarm( $s, expr$ )  
allocatemem( $\{s\}$ )  
alias( $new, old$ )  
install( $f, code, \{gpf\}, \{lib\}$ )  
system( $a$ )  
extern( $a$ )  
externstr( $a$ )  
getenv("VAR")  
strexpend( $x$ )

parapply( $f, x$ )  
pareval( $f$ )  
parselect( $f, A, \{flag\}$ )  
parsum( $i = a, b, expr$ )  
parvector( $n, i, \{expr\}$ )  
parfor( $i = a, \{b\}, f, \{r\}, \{f_2\}$ )  
parforeach( $v, x, f, \{r\}, \{f_2\}$ )  
parforprime( $p = a, \{b\}, f, \{r\}, \{f_2\}$ )  
parforprimestep( $p = a, \{b\}, q, f, \{r\}, \{f_2\}$ )  
parforstep( $i = a, \{b\}, s, f, \{r\}, \{f_2\}$ )  
parforvec( $X = v, f, \{r\}, \{f_2\}, \{flag\}$ )  
export( $x$ )  
exportall()  
unexport( $x$ )  
unexportall()

matsize( $x$ )  
 $x * y$   
matmultodiagonal( $x, y$ )  
concat( $x, \{y\}$ )  
vecextract( $x, y, \{z\}$ )  
 $x\sim$ , mattranspose( $x$ )  
matadjoint( $x$ )  
mateigen( $x$ )  
charpoly( $x$ ), minpoly( $x$ )  
trace( $x$ ), matdet( $x$ )  
matpermanent( $x$ )  
matfrobenius( $x$ )  
matqr( $x$ )  
mathouseholder( $Q, v$ )

$[g(x) \mid x \leftarrow v, f(x)]$   
 $[x \mid x \leftarrow v, f(x)]$   
 $[g(x) \mid x \leftarrow v]$   
vector( $n, \{i\}, \{expr\}$ )  
vectorv( $n, \{i\}, \{expr\}$ )  
vectorsmall( $n, \{i\}, \{expr\}$ )

$[c, c \cdot x, \dots, c \cdot x^n]$   
 $[1, 2^x, \dots, n^x]$   
matrix  $1 \leq i \leq m, 1 \leq j \leq n$   
define matrix by blocks  
diagonal matrix with diagonal  $x$   
is  $x$  diagonal?  
 $x \cdot \text{matdiagonal}(d)$   
 $n \times n$  identity matrix

Hessenberg form of square matrix  $x$   
 $n \times n$  Hilbert matrix  $H_{ij} = (i + j - 1)^{-1}$   
 $n \times n$  Pascal triangle  
companion matrix to polynomial  $x$   
Sylvester matrix of  $x$  and  $y$

## Gaussian elimination

kernel of matrix  $x$   
intersection of column spaces of  $x$  and  $y$   
solve  $MX = B$  ( $M$  invertible)  
one sol of  $M * X = B$   
basis for image of matrix  $x$   
columns of  $x$  *not* in **matimage**  
supplement columns of  $x$  to get basis  
rows, cols to extract invertible matrix  
rank of the matrix  $x$   
solve  $MX = B \bmod D$   
image mod  $D$   
kernel mod  $D$   
inverse mod  $D$   
determinant mod  $D$

## Lattices & Quadratic Forms

### Quadratic forms

evaluate  ${}^t x Q y$   
evaluate  ${}^t x Q x$   
signature of quad form  ${}^t y * x * y$   
decomp into squares of  ${}^t y * x * y$   
eigenvalues/vectors for real symmetric  $x$   
Cholesky decomposition of  $x$

### HNF and SNF

upper triangular Hermite Normal Form  
HNF of  $x$  where  $d$  is a multiple of  $\det(x)$   
multiple of  $\det(x)$   
HNF of  $(x \mid \text{diagonal}(D))$   
elementary divisors of  $x$   
 $q$ -rank from elementary divisors  
elementary divisors of  $\mathbf{Z}[a]/(f'(a))$   
integer kernel of  $x$   
**Z**-module  $\leftrightarrow$  **Q**-vector space

### Lattices

LLL-algorithm applied to columns of  $x$   
... for Gram matrix of lattice  
find up to  $m$  sols of  $qfeval(x, y) \leq b$   
... up to  $m$  closest vectors to  $t$   
 $v, v[i] :=$  number of  $y$  s.t.  $qfeval(x, y) = i$   
perfection rank of  $x$   
find isomorphism between  $q$  and  $Q$

Based on an earlier version by Joseph H. Silverman  
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Send comments and corrections to (Karim.Belabas@math.u-bordeaux.fr)

powers( $x, n, \{c = 1\}$ )  
dirpowers( $n, x$ )  
matrix( $m, n, \{i\}, \{j\}, \{expr\}$ )  
matconcat( $B$ )  
matdiagonal( $x$ )  
matisdiagonal( $x$ )  
matmultiagonal( $x, d$ )  
matid( $n$ )  
mathess( $x$ )  
mathilbert( $n$ )  
matpascal( $n - 1$ )  
matcompanion( $x$ )  
polsylvestermatrix( $x, y$ )

matker( $x, \{flag\}$ )  
matintersect( $x, y$ )  
matsolve( $M, B$ )  
matinverseimage( $M, B$ )  
matimage( $x$ )  
matimagecompl( $x$ )  
matsupplement( $x$ )  
matindexrank( $x$ )  
matrank( $x$ )  
matsolvenmod( $M, D, B$ )  
matimagemod( $M, D$ )  
matkermod( $M, D$ )  
matinvmod( $M, D$ )  
matdetmod( $M, D$ )

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precompute for isomorphism test with  $q$     `qfisominit(q)`  
 automorphism group of  $q$     `qfauto(q)`  
 convert `qfauto` for GAP/Magma    `qfautoexport(G, {flag})`  
 orbits of  $V$  under  $G \subset \text{GL}(V)$     `qforbits(G, V)`

## Polynomials & Rational Functions

all defined polynomial variables    `variables()`  
 get var. of highest priority (higher than  $v$ )    `varhigher(name, {v})`  
 ... of lowest priority (lower than  $v$ )    `varlower(name, {v})`

### Coefficients, variables and basic operators

degree of  $f$     `poldegree(f)`  
 coef. of degree  $n$  of  $f$ , leading coef.    `polcoef(f, n)`, `pollead`  
 main variable / all variables in  $f$     `variable(f)`, `variables(f)`  
 replace  $x$  by  $y$  in  $f$     `subst(f, x, y)`  
 evaluate  $f$  replacing vars by their value    `eval(f)`  
 replace polynomial expr.  $T(x)$  by  $y$  in  $f$     `substpol(f, T, y)`  
 replace  $x_1, \dots, x_n$  by  $y_1, \dots, y_n$  in  $f$     `substvec(f, x, y)`  
 $f \in A[x]$ ; reciprocal polynomial  $x^{\deg f} f\left(\frac{1}{x}\right)$     `polrecip(f)`  
 gcd of coefficients of  $f$     `content(f)`  
 derivative of  $f$  w.r.t.  $x$     `deriv(f, {x})`  
 ...  $n$ -th derivative of  $f$     `derivn(f, n, {x})`  
 formal integral of  $f$  w.r.t.  $x$     `intformal(f, {x})`  
 formal sum of  $f$  w.r.t.  $x$     `sumformal(f, {x})`

### Constructors & Special Polynomials

interpolation polynomial at  $(x[1], y[1]), \dots, (x[n], y[n])$ , evaluated at  $t$ , with error estimate  $e$     `polinterpolate(x, {y}, {t}, {&e})`  
 monic polynomial from roots  $r$     `polfromroots(r)`  
 $T_n/U_n, H_n$     `polchebyshev(n)`, `polhermite(n)`  
 $P_n, L_n^{(\alpha)}$     `pollegendre(n)`, `pollaguerre(n, a)`  
 $n$ -th cyclotomic polynomial  $\Phi_n$     `polcyclo(n)`  
 return  $n$  if  $f = \Phi_n$ , else 0    `poliscyclo(f)`  
 is  $f$  a product of cyclotomic polynomials?    `poliscycloprod(f)`  
 Zagier's polynomial of index  $(n, m)$     `polzagier(n, m)`

### Resultant, elimination

discriminant of polynomial  $f$     `poldisc(f)`  
 find factors of `poldisc(f)`    `poldiscfactors(f)`  
 resultant  $R = \text{Res}_v(f, g)$     `polresultant(f, g, {v})`  
 $[u, v, R], xu + yv = \text{Res}_v(f, g)$     `polresultantext(x, y, {v})`  
 solve Thue equation  $f(x, y) = a$     `thue(t, a, {sol})`  
 initialize  $t$  for Thue equation solver    `thueinit(f)`

### Roots and Factorization (Complex/Real)

complex roots of  $f$     `polroots(f)`  
 bound complex roots of  $f$     `polrootsbound(f)`  
 number of real roots of  $f$  (in  $[a, b]$ )    `polsturm(f, {[a, b]})`  
 real roots of  $f$  (in  $[a, b]$ )    `polrootsreal(f, {[a, b]})`  
 complex embeddings of `t.POLMOD z`    `conjvec(z)`

### Roots and Factorization (Finite fields)

factor  $f$  mod  $p$ , roots    `factormod(f, p)`, `polrootsmod`  
 factor  $f$  over  $\mathbf{F}_p[x]/(T)$ , roots    `factormod(f, [T, p])`, `polrootsmod`  
 squarefree factorization of  $f$  in  $\mathbf{F}_q[x]$     `factormodSQF(f, {D})`  
 distinct degree factorization of  $f$  in  $\mathbf{F}_q[x]$     `factormodDDF(f, {D})`  
 factor  $n$ -th cyclotomic pol.  $\Phi_n$  mod  $p$     `factormodcyclo(n, p)`

### Roots and Factorization ( $p$ -adic fields)

factor  $f$  over  $\mathbf{Q}_p$ , roots    `factorpadic(f, p, r)`, `polrootspadic`  
 $p$ -adic root of  $f$  congruent to  $a$  mod  $p$     `padicappr(f, a)`  
 Newton polygon of  $f$  for prime  $p$     `newtonpoly(f, p)`  
 Hensel lift  $A/\text{lc}(A) = \prod_i B[i] \pmod{p^e}$     `polhensellift(A, B, p, e)`

$T = \prod (x - z_i) \mapsto \prod (x - \omega(z_i)) \in \mathbf{Z}_p[x]$     `polteichmuller(T, p, e)`  
 extensions of  $\mathbf{Q}_p$  of degree  $N$     `padicfields(p, N)`

### Roots and Factorization (Miscellaneous)

symmetric powers of roots of  $f$  up to  $n$     `polysym(f, n)`  
 Graeffe transform of  $f$ ,  $g(x^2) = f(x)f(-x)$     `polgraeffe(f)`  
 factor  $f$  over coefficient field    `factor(f)`  
 cyclotomic factors of  $f \in \mathbf{Q}[X]$     `polcyclofactors(f)`

### Finite Fields

A finite field is encoded by any element (`t_FFELT`).  
 find irreducible  $T \in \mathbf{F}_p[x]$ ,  $\deg T = n$     `ffinit(p, n, {x})`  
 Create  $t$  in  $\mathbf{F}_q \simeq \mathbf{F}_p[t]/(T)$     `t = ffgent(T, 't)`  
 ... indirectly, with implicit  $T$     `t = ffgent(q, 't); T = t.mod`  
 map  $m$  from  $\mathbf{F}_q \ni a$  to  $\mathbf{F}_{q^k} \ni b$     `m = ffembed(a, b)`  
 build  $K = \mathbf{F}_q[x]/(P)$  extending  $\mathbf{F}_q \ni a$ ,    `ffextend(a, P)`  
 evaluate map  $m$  on  $x$     `ffmap(m, x)`  
 inverse map of  $m$     `ffinvm(m)`  
 compose maps  $m \circ n$     `ffcompom(m, n)`  
 $x$  as polmod over codomain of map  $m$     `ffmaprel(m, x)`  
 $F^n$  over  $\mathbf{F}_q \ni a$     `fffrobenius(a, n)`  
 $\#\{\text{monic irred. } T \in \mathbf{F}_q[x], \deg T = n\}$     `ffnbirred(q, n)`

### Formal & $p$ -adic Series

truncate power series or  $p$ -adic number    `truncate(x)`  
 valuation of  $x$  at  $p$     `valuation(x, p)`

### Dirichlet and Power Series

Taylor expansion around 0 of  $f$  w.r.t.  $x$     `taylor(f, x)`  
 Laurent series of closure  $F$  up to  $x^k$     `laurentseries(f, k)`  
 $\sum a_k b_k t^k$  from  $\sum a_k t^k$  and  $\sum b_k t^k$     `serconvol(a, b)`  
 $f = \sum a_k t^k$  from  $\sum (a_k/k!) t^k$     `serlaplace(f)`  
 reverse power series  $F$  so  $F(f(x)) = x$     `serreverse(f)`  
 remove terms of degree  $< n$  in  $f$     `serchop(f, n)`  
 Dirichlet series multiplication / division    `dirmul, dirdiv(x, y)`  
 Dirichlet Euler product ( $b$  terms)    `direuler(p = a, b, expr)`

### Transcendental and $p$ -adic Functions

real, imaginary part of  $x$     `real(x)`, `imag(x)`  
 absolute value, argument of  $x$     `abs(x)`, `arg(x)`  
 square/ $n$ th root of  $x$     `sqrt(x)`, `sqrtn(x, n, {&z})`  
 all  $n$ -th roots of 1    `rootsof1(n)`  
 FFT of  $[f_0, \dots, f_{n-1}]$     `w = fftinit(n)`, `fft/fftinw(w, f)`  
 trig functions    `sin, cos, tan, cotan, sinc`  
 inverse trig functions    `asin, acos, atan`  
 hyperbolic functions    `sinh, cosh, tanh, cotanh`  
 inverse hyperbolic functions    `asinh, acosh, atanh`  
 $\log(x)$ ,  $\log(1+x)$ ,  $e^x$ ,  $e^x - 1$     `log, log1p, exp, expm1`  
 Euler  $\Gamma$  function,  $\log \Gamma$ ,  $\Gamma'/\Gamma$     `gamma, lngamma, psi`  
 half-integer gamma function  $\Gamma(n+1/2)$     `gammah(n)`  
 Riemann's zeta  $\zeta(s) = \sum n^{-s}$     `zeta(s)`  
 $\sum_{1 \leq n \leq N} n^s$     `dirpowersum(N, s)`  
 Hurwitz's  $\zeta(s, x) = \sum (n+x)^{-s}$     `zetahurwitz(s, x)`  
 Lerch  $\Phi(z, s, x) = \sum z^n (n+x)^{-s}$     `lerchphi(z, s, x)`  
 Lerch  $L(s, x, t) = \Phi(e^{2i\pi t}, s, x)$     `lerchzeta(s, x, t)`  
 multiple zeta value (MZV),  $\zeta(s_1, \dots, s_k)$     `zetamult(s, {T})`  
 all MZVs for weight  $\sum s_i = n$     `zetamultall(n)`  
 convert MZV id to  $[s_1, \dots, s_k]$     `zetamultconvert(f, {flag})`  
 MZV dual sequence    `zetamultdual(s)`  
 multiple polylog  $Li_{s_1, \dots, s_k}(z_1, \dots, z_k)$     `polylogmult(s, z)`

incomplete  $\Gamma$  function ( $y = \Gamma(s)$ )    `incgam(s, x, {y})`  
 complementary incomplete  $\Gamma$     `incgamc(s, x)`  
 $\int_x^\infty e^{-t} dt/t$ ,  $(2/\sqrt{\pi}) \int_x^\infty e^{-t^2} dt$     `eint1, erfc`  
 elliptic integral of 1st and 2nd kind    `ellK(k)`, `ellE(k)`  
 dilogarithm of  $x$     `dilog(x)`  
 $m$ -th polylogarithm of  $x$     `polylog(m, x, {flag})`  
 $U$ -confluent hypergeometric function    `hyperu(a, b, u)`  
 Hypergeometric  ${}_pF_q(A, B; z)$     `hypergeom(A, B, z)`  
 Bessel  $J_n(x)$ ,  $J_{n+1/2}(x)$     `besselj(n, x)`, `besseljh(n, x)`  
 Bessel  $I_\nu, K_\nu, H_\nu^1, H_\nu^2, Y_\nu$     `(bessel)i, k, h1, h2, y`  
 $k$ -th zero of  $J_\nu(x)$     `besseljzero(nu, {k = 1})`  
 $k$ -th zero of  $Y_\nu(x)$     `besselyzero(nu, {k = 1})`  
 Airy functions  $A_i(x)$ ,  $B_i(x)$     `airy(x)`  
 Lambert  $W$ :  $x$  s.t.  $xe^x = y$     `lambertw(y)`  
 Teichmuller character of  $p$ -adic  $x$     `teichmuller(x)`

## Iterations, Sums & Products

### Numerical integration for meromorphic functions

Behaviour at endpoint for Double Exponential (DE) methods: either a scalar ( $a \in \mathbf{C}$ , regular) or  $\pm\infty$  (decreasing at least as  $x^{-2}$ ) or

$(x-a)^{-\alpha}$ singularity	<code>[a, a]</code>
exponential decrease $e^{-\alpha x }$	<code>[±∞, a]</code> , $\alpha > 0$
slow decrease $ x ^\alpha$	<code>... α &lt; -1</code>
oscillating as $\cos(kx)$	$\alpha = k\mathbf{I}$ , $k > 0$
oscillating as $\sin(kx)$	$\alpha = -k\mathbf{I}$ , $k > 0$

numerical integration    `intnum(x = a, b, f, {T})`  
 weights  $T$  for intnum    `intnuminit(a, b, {m})`  
 weights  $T$  incl. kernel  $K$     `intfuncinit(t = a, b, K, {m})`  
 integrate  $(2i\pi)^{-1} f$  on circle  $|z-a|=R$     `intcirc(x = a, R, f, {T})`

### Other integration methods

$n$ -point Gauss-Legendre    `intnumgauss(x = a, b, f, {n})`  
 weights for  $n$ -point Gauss-Legendre    `intnumgaussinit({n})`  
 quasi-periodic function, period  $2H$     `intnumosc(x = a, f, H)`  
 Romberg (low accuracy)    `intnumromb(x = a, b, f, {flag})`

### Numerical summation

sum of series  $f(n)$ ,  $n \geq a$  (low accuracy)    `suminf(n = a, expr)`  
 sum of alternating/positive series    `sumalt, sumpos`  
 sum of series using Euler-Maclaurin    `sumnum(n = a, f, {T})`  
 ... Sidi summation    `sumnumsidi(n = a, f)`  
 $\sum_{n \geq a} F(n)$ ,  $F$  rational function    `sumnumrat(F, a)`  
 $\dots \sum_{p \geq a} F(p^s)$     `sumeulerrat(F, {s = 1}, {a = 2})`  
 weights for `sumnum`,  $a$  as in DE    `sumnuminit({∞, a})`  
 sum of series by Monien summation    `sumnummonien(n = a, f, {T})`  
 weights for `sumnummonien`    `sumnummonieninit({∞, a})`  
 sum of series using Abel-Plana    `sumnumap(n = a, f, {T})`  
 weights for `sumnumap`,  $a$  as in DE    `sumnumapinit({∞, a})`  
 sum of series using Lagrange    `sumnumlagrange(n = a, f, {T})`  
 weights for `sumnumlagrange`    `sumnumlagrangeinit`

### Products

product  $a \leq X \leq b$ , initialized at  $x$     `prod(X = a, b, expr, {x})`  
 product over primes  $a \leq X \leq b$     `prodeuler(X = a, b, expr)`  
 infinite product  $a \leq X \leq \infty$     `prodinf(X = a, expr)`  
 $\prod_{n \geq a} F(n)$ ,  $F$  rational function    `prodnumrat(F, a)`  
 $\prod_{p \geq a} F(p^s)$     `prodeulerrat(F, {s = 1}, {a = 2})`

## Other numerical methods

real root of  $f$  in  $[a, b]$ ; bracketed root `solve( $X = a, b, f$ )`  
 ... interval splitting, step  $s$  `solvestep( $X = a, b, s, f, \{flag = 0\}$ )`  
 limit of  $f(t)$ ,  $t \rightarrow \infty$  `limitnum( $f, \{\alpha\}$ )`  
 asymptotic expansion of  $f$  (rational) `asymnum( $f, \{\alpha\}$ )`  
 ...  $N + 1$  terms as floats `asymnumraw( $f, N, \{\alpha\}$ )`  
 numerical derivation w.r.t  $x$ :  $f'(a)$  `derivnum( $x = a, f$ )`  
 evaluate continued fraction  $F$  at  $t$  `contfraceval( $F, t, \{L\}$ )`  
 power series to cont. fraction ( $L$  terms) `contfracinit( $S, \{L\}$ )`  
 Padé approximant (deg. denom.  $\leq B$ ) `bestapprPade( $S, \{B\}$ )`

## Elementary Arithmetic Functions

vector of binary digits of  $|x|$  `binary( $x$ )`  
 bit number  $n$  of integer  $x$  `bittest( $x, n$ )`  
 Hamming weight of integer  $x$  `hammingweight( $x$ )`  
 digits of integer  $x$  in base  $B$  `digits( $x, \{B = 10\}$ )`  
 sum of digits of integer  $x$  in base  $B$  `sumdigits( $x, \{B = 10\}$ )`  
 integer from digits `fromdigits( $v, \{B = 10\}$ )`  
 ceiling/floor/fractional part `ceil, floor, frac`  
 round  $x$  to nearest integer `round( $x, \{\&e\}$ )`  
 truncate  $x$  `truncate( $x, \{\&e\}$ )`  
 gcd/LCM of  $x$  and  $y$  `gcd( $x, y$ ), lcm( $x, y$ )`  
 gcd of entries of a vector/matrix `content( $x$ )`

## Primes and Factorization

extra prime table `addprimes()`  
 add primes in  $v$  to prime table `addprimes( $v$ )`  
 remove primes from prime table `removeprimes( $v$ )`  
 Chebyshev  $\pi(x)$ ,  $n$ -th prime  $p_n$  `primepi( $x$ ), prime( $n$ )`  
 vector of first  $n$  primes `primes( $n$ )`  
 smallest prime  $\geq x$  `nextprime( $x$ )`  
 largest prime  $\leq x$  `preprime( $x$ )`  
 factorization of  $x$  `factor( $x, \{lim\}$ )`  
 ... selecting specific algorithms `factorint( $x, \{flag = 0\}$ )`  
 $n = df^2$ ,  $d$  squarefree/fundamental `core( $n, \{fl\}$ ), coredisc`  
 certificate for (prime)  $N$  `primecert( $N$ )`  
 verifies a certificate  $c$  `primecertisvalid( $c$ )`  
 convert certificate to Magma/PRIMO `primecertexport`  
 recover  $x$  from its factorization `factorback( $f, \{e\}$ )`  
 $x \in \mathbf{Z}$ ,  $|x| \leq X$ ,  $\gcd(N, P(x)) \geq N$  `zncoppersmith( $P, N, X, \{B\}$ )`  
 divisors of  $N$  in residue class  $r$  mod  $s$  `divisorslensstra( $N, r, s$ )`

## Divisors and multiplicative functions

number of prime divisors  $\omega(n)$  /  $\Omega(n)$  `omega( $n$ ), bigomega`  
 divisors of  $n$  / number of divisors  $\tau(n)$  `divisors( $n$ ), numdiv`  
 sum of ( $k$ -th powers of) divisors of  $n$  `sigma( $n, \{k\}$ )`  
 Möbius  $\mu$ -function `moebias( $x$ )`  
 Ramanujan's  $\tau$ -function `ramanujantau( $x$ )`

## Combinatorics

factorial of  $x$  `x!` or `factorial( $x$ )`  
 binomial coefficient  $\binom{x}{k}$  `binomial( $x, \{k\}$ )`  
 Bernoulli number  $B_n$  as real/rational `bernreal( $n$ ), bernfrac`  
 $[B_0, B_2, \dots, B_{2k}]$  `bernvec( $k$ )`  
 Bernoulli polynomial  $B_n(x)$  `bernpol( $n, \{x\}$ )`  
 Euler numbers `eulerfrac, eulerreal, eulervec`  
 Euler polynomial  $E_n(x)$  `eulerpol( $n, \{x\}$ )`  
 Eulerian polynomial  $A_n(x)$  `eulerianpol`  
 Fibonacci number  $F_n$  `fibonacci( $n$ )`  
 Harmonic number  $H_{n,r} = 1^{-r} + \dots + n^{-r}$  `harmonic( $n, r$ )`  
 Stirling numbers  $s(n, k)$  and  $S(n, k)$  `stirling( $n, k, \{flag\}$ )`

# Pari-GP reference card

(PARI-GP version 2.17.1)

number of partitions of  $n$  `numbpart( $n$ )`  
 $k$ -th permutation on  $n$  letters `numtoperm( $n, k$ )`  
 ... index  $k$  of permutation  $v$  `permtionum( $v$ )`  
 order of permutation  $p$  `permorder( $p$ )`  
 signature of permutation  $p$  `permsign( $p$ )`  
 cyclic decomposition of permutation  $p$  `permcycles( $p$ )`  
**Multiplicative groups ( $\mathbf{Z}/N\mathbf{Z}$ )\*,  $\mathbf{F}_q^*$**   
 Euler  $\phi$ -function `eulerphi( $x$ )`  
 multiplicative order of  $x$  (divides  $o$ ) `znorder( $x, \{o\}$ ), fforder`  
 primitive root mod  $q / x$ .mod `znprimroot( $q$ ), fprimroot( $x$ )`  
 structure of  $(\mathbf{Z}/n\mathbf{Z})^*$  `znstar( $n$ )`  
 discrete logarithm of  $x$  in base  $g$  `znlog( $x, g, \{o\}$ ), fflag`  
 Kronecker-Legendre symbol  $(\frac{x}{y})$  `kronecker( $x, y$ )`  
 quadratic Hilbert symbol (at  $p$ ) `hilbert( $x, y, \{p\}$ )`  
**Euclidean algorithm, continued fractions**  
 CRT: solve  $z \equiv x$  and  $z \equiv y$  `chinese( $x, y$ )`  
 minimal  $u, v$  so  $xu + yv = \gcd(x, y)$  `gcdext( $x, y$ )`  
 half-gcd algorithm `halfgcd( $x, y$ )`  
 continued fraction of  $x$  `contfrac( $x, \{b\}, \{lmax\}$ )`  
 last convergent of continued fraction  $x$  `contfracpnqn( $x$ )`  
 rational approximation to  $x$  (den.  $\leq B$ ) `bestappr( $x, \{B\}$ )`  
 recognize  $x \in \mathbf{C}$  as polmod mod  $T \in \mathbf{Z}[X]$  `bestapprnf( $x, T$ )`

## Miscellaneous

integer square /  $n$ -th root of  $x$  `sqrtnint( $x$ ), sqrtint( $x, n$ )`  
 largest integer  $e$  s.t.  $b^e \leq x$ ,  $e = \lfloor \log_b(x) \rfloor$  `logint( $x, b, \{\&z\}$ )`

## Characters

Let  $cyc = [d_1, \dots, d_k]$  represent an abelian group  $G = \oplus (\mathbf{Z}/d_j\mathbf{Z}) \cdot g_j$  or any structure  $G$  affording a .cyc method; e.g. `znstar( $q, 1$ )` for Dirichlet characters. A character  $\chi$  is coded by  $[c_1, \dots, c_k]$  such that  $\chi(g_j) = e(n_j/d_j)$ .  
 $\chi \cdot \psi$ ;  $\chi^{-1}$ ;  $\chi \cdot \psi^{-1}$ ;  $\chi^k$  `charm, charconj, chardiv, charpow`  
 order of  $\chi$  `charorder( $cyc, \chi$ )`  
 kernel of  $\chi$  `charker( $cyc, \chi$ )`  
 $\chi(x)$ ,  $G$  a GP group structure `chareval( $G, \chi, x, \{z\}$ )`  
 Galois orbits of characters `chargalois( $G$ )`

## Dirichlet Characters

initialize  $G = (\mathbf{Z}/q\mathbf{Z})^*$  `G = znstar( $q, 1$ )`  
 convert datum  $D$  to  $[G, \chi]$  `znchar( $D$ )`  
 is  $\chi$  odd? `zncharisodd( $G, \chi$ )`  
 real  $\chi \rightarrow$  Kronecker symbol  $(D/\cdot)$  `znchartokronecker( $G, \chi$ )`  
 conductor of  $\chi$  `zncharconductor( $G, \chi$ )`  
 $[G_0, \chi_0]$  primitive attached to  $\chi$  `znchartoprimitive( $G, \chi$ )`  
 induce  $\chi \in \hat{G}$  to  $\mathbf{Z}/N\mathbf{Z}$  `zncharinduce( $G, \chi, N$ )`  
 $\chi^p$  `znchardecompose( $G, \chi, p$ )`  
 $\prod_p |(Q, N) \chi^p$  `znchardecompose( $G, \chi, Q$ )`  
 complex Gauss sum  $G_a(\chi)$  `znchargauss( $G, \chi$ )`

## Conrey labelling

Conrey label  $m \in (\mathbf{Z}/q\mathbf{Z})^* \rightarrow$  character `znconreychar( $G, m$ )`  
 character  $\rightarrow$  Conrey label `znconreyexp( $G, \chi$ )`  
 log on Conrey generators `znconreylog( $G, m$ )`  
 conductor of  $\chi$  ( $\chi_0$  primitive) `znconreyconductor( $G, \chi, \{\chi_0\}$ )`

## True-False Tests

is  $x$  the disc. of a quadratic field? `isfundamental( $x$ )`  
 is  $x$  a prime? `isprime( $x$ )`  
 is  $x$  a strong pseudo-prime? `ispseudoprime( $x$ )`  
 is  $x$  square-free? `issquarefree( $x$ )`  
 is  $x$  a square? `issquare( $x, \{\&n\}$ )`  
 is  $x$  a perfect power? `ispower( $x, \{k\}, \{\&n\}$ )`  
 is  $x$  a perfect power of a prime? ( $x = p^n$ ) `isprimepower( $x, \&n$ )`  
 ... of a pseudoprime? `ispseudoprimepower( $x, \&n$ )`  
 is  $x$  powerful? `ispowerful( $x$ )`  
 is  $x$  a totient? ( $x = \varphi(n)$ ) `istotient( $x, \{\&n\}$ )`  
 is  $x$  a polygonal number? ( $x = P(s, n)$ ) `ispolygonal( $x, s, \{\&n\}$ )`  
 is  $pol$  irreducible? `polisirreducible( $pol$ )`

## Graphic Functions

crude graph of  $expr$  between  $a$  and  $b$  `plot( $X = a, b, expr$ )`  
**High-resolution plot** (immediate plot)  
 plot  $expr$  between  $a$  and  $b$  `plotth( $X = a, b, expr, \{flag\}, \{n\}$ )`  
 plot points given by lists  $lx, ly$  `plotthraw( $lx, ly, \{flag\}$ )`  
 terminal dimensions `plotsizes()`

## Rectwindow functions

init window  $w$ , with size  $x, y$  `plotinit( $w, x, y$ )`  
 erase window  $w$  `plotkill( $w$ )`  
 copy  $w$  to  $w_2$  with offset  $(dx, dy)$  `plotcopy( $w, w_2, dx, dy$ )`  
 clips contents of  $w$  `plotclip( $w$ )`  
 scale coordinates in  $w$  `plotscale( $w, x_1, x_2, y_1, y_2$ )`  
`plotth` in  $w$  `plotrecth( $w, X = a, b, expr, \{flag\}, \{n\}$ )`  
`plotthraw` in  $w$  `plotrecthraw( $w, data, \{flag\}$ )`  
 draw window  $w_1$  at  $(x_1, y_1), \dots$  `plotdraw([[ $w_1, x_1, y_1$ ], ...])`

## Low-level Rectwindow Functions

set current drawing color in  $w$  to  $c$  `plotcolor( $w, c$ )`  
 current position of cursor in  $w$  `plotcursor( $w$ )`  
 write  $s$  at cursor's position `plotstring( $w, s$ )`  
 move cursor to  $(x, y)$  `plotmove( $w, x, y$ )`  
 move cursor to  $(x + dx, y + dy)$  `plotrmove( $w, dx, dy$ )`  
 draw a box to  $(x_2, y_2)$  `plotbox( $w, x_2, y_2$ )`  
 draw a box to  $(x + dx, y + dy)$  `plotrbox( $w, dx, dy$ )`  
 draw polygon `plotlines( $w, lx, ly, \{flag\}$ )`  
 draw points `plotpoints( $w, lx, ly$ )`  
 draw ellipse `plotarc( $w, lx, ly, \{flag\}$ )`  
 draw line to  $(x + dx, y + dy)$  `plotrline( $w, dx, dy$ )`  
 draw point  $(x + dx, y + dy)$  `plotrpoint( $w, dx, dy$ )`

## Convert to Postscript or Scalable Vector Graphics

The format  $f$  is either "ps" or "svg".  
 as `plotth` `plotlexport( $f, X = a, b, expr, \{flag\}, \{n\}$ )`  
 as `plotthraw` `plotthrawexport( $f, lx, ly, \{flag\}$ )`  
 as `plotdraw` `plotlexport( $f, [[ $w_1, x_1, y_1$ ], ...])$`

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