

Pari-GP reference card

(PARI-GP version 2.13.3)

Note: optional arguments are surrounded by braces {}.
To start the calculator, type its name in the terminal: **gp**
To exit **gp**, type **quit**, **\q**, or **<C-D>** at prompt.

Help

describe function	?function
extended description	??keyword
list of relevant help topics	???pattern
name of GP-1.39 function f in GP-2.*	whatnow(f)

Input/Output

previous result, the result before	%, %`, %`` , etc.
n -th result since startup	% n
separate multiple statements on line	;
extend statement on additional lines	\
extend statements on several lines	{seq ₁ ; seq ₂ ;}
comment	/* ... */
one-line comment, rest of line ignored	\\ ...

Metacommands & Defaults

set default d to val	default({ d },{ val })
toggle timer on/off	#
print time for last result	##
print defaults	\d
set debug level to n	\g n
set memory debug level to n	\gm n
set n significant digits / bits	\p n , \pb n
set n terms in series	\ps n
quit GP	\q
print the list of PARI types	\t
print the list of user-defined functions	\u
read file into GP	\r <i>filename</i>

Debugger / break loop

get out of break loop	break or <C-D>
go up/down n frames	dbg_up({ n }), dbg_down
set break point	breakpoint()
examine object o	dbg_x(o)
current error data	dbg_err()
number of objects on heap and their size	getheap()
total size of objects on PARI stack	getstack()

PARI Types & Input Formats

t_INT. Integers; hex, binary	± 31 ; $\pm 0x1F$, $\pm 0b101$
t_REAL. Reals	± 3.14 , 6.022 E23
t_INTMOD. Integers modulo m	Mod(n , m)
t_FRAC. Rational Numbers	n/m
t_FFELT. Elt in finite field \mathbf{F}_q	ffgen(q , 't)
t_COMPLEX. Complex Numbers	$x + y * I$
t_PADIC. p -adic Numbers	$x + 0(p^{-k})$
t_QUAD. Quadratic Numbers	$x + y * \text{quadgen}(D, \{ 'w \})$
t_POLMOD. Polynomials modulo g	Mod(f , g)
t_POL. Polynomials	$a * x^n + \dots + b$
t_SER. Power Series	$f + 0(x^{-k})$
t_RFRAC. Rational Functions	f/g
t_QFI/t_QFR. Imag/Real binary quad. form	Qfb($a, b, c, \{d\}$)
t_VEC/t_COL. Row/Column Vectors	$[x, y, z]$, $[x, y, z] \sim$
t_VEC integer range	$[1..10]$

t_VECSMALL. Vector of small ints	Vecsmall($[x, y, z]$)
t_MAT. Matrices	$[a, b; c, d]$
t_LIST. Lists	List($[x, y, z]$)
t_STR. Strings	"abc"
t_INFINITY. $\pm\infty$	+oo, -oo

Reserved Variable Names

$\pi \approx 3.14$, $\gamma \approx 0.57$, $C \approx 0.91$, $I = \sqrt{-1}$	Pi, Euler, Catalan, I
Landau's big-oh notation	O

Information about an Object, Precision

PARI type of object x	type(x)
length of x / size of x in memory	# x , sizebyte(x)
real precision / bit precision of x	precision(x), bitprecision(x)
p -adic, series prec. of x	padicprec(x, p), serprec(x, v)
current dynamic precision	getlocalprec, getlocalbitprec

Operators

basic operations	+, -, *, /, ^, sqr
$i \leftarrow i+1$, $i \leftarrow i-1$, $i \leftarrow i*j$, ...	i++, i--, i*=j, ...
Euclidean quotient, remainder	$x \backslash y$, $x \backslash y$, $x \% y$, divrem(x, y)
shift x left or right n bits	$x \ll n$, $x \gg n$ or shift($x, \pm n$)
multiply by 2^n	shiftmul(x, n)
comparison operators	<=, <, >=, >, ==, !=, ==, lex, cmp
boolean operators (or, and, not)	, &&, !
bit operations	bitand, bitneg, bitor, bitxor, bitnegimply
maximum/minimum of x and y	max(x, y), min(x, y)
sign of x (gives $-1, 0, 1$)	sign(x)
binary exponent of x	exponent(x)
derivative of f , 2nd derivative, etc.	f' , f'' , ...
differential operator	diffop($f, v, d, \{n = 1\}$)
quote operator (formal variable)	'x
assignment	x = value
simultaneous assignment $x \leftarrow v[1]$, $y \leftarrow v[2]$	[x,y] = v

Select Components

Caveat: components start at index $n = 1$.

n -th component of x	component(x, n)
n -th component of vector/list x	$x[n]$
components $a, a + 1, \dots, b$ of vector x	$x[a..b]$
(m, n) -th component of matrix x	$x[m, n]$
row m or column n of matrix x	$x[m,]$, $x[, n]$
numerator/denominator of x	numerator(x), denominator(x)

Random Numbers

random integer/prime in $[0, N[$	random(N), randomprime(N)
get/set random seed	getrand, setrand(s)

Conversions

to vector, matrix, vec. of small ints	Col/Vec, Mat, Vecsmall
to list, set, map, string	List, Set, Map, Str
create $(x \bmod y)$	Mod(x, y)
make x a polynomial of v	Pol($x, \{v\}$)
variants of Pol <i>et al.</i> , in reverse order	Polrev, Vecrev, Colrev
make x a power series of v	Ser($x, \{v\}$)
convert x to simplest possible type	simplify(x)
object x with real precision n	precision(x, n)
object x with bit precision n	bitprecision(x, n)
set precision to p digits in dynamic scope	localprec(p)
set precision to p bits in dynamic scope	localbitprec(p)

Character strings

convert to TeX representation	strtex(x)
string from bytes / from format+args	strchr, sprintf
split string / join strings	strsplit, strjoin
convert time t ms. to h, m, s, ms format	strtime(t)

Conjugates and Lifts

conjugate of a number x	conj(x)
norm of x , product with conjugate	norm(x)
L^p norm of x (L^∞ if no p)	normlp($x, \{p\}$)
square of L^2 norm of x	norml2(x)
lift of x from Mods and p -adics	lift, centerlift(x)
recursive lift	liftall
lift all t_INT and t_PADIC (\rightarrow t_INT)	liftint
lift all t_POLMOD (\rightarrow t_POL)	liftpol

Lists, Sets & Maps

Sets (= row vector with strictly increasing entries w.r.t. cmp)

intersection of sets x and y	setintersect(x, y)
set of elements in x not belonging to y	setminus(x, y)
union of sets x and y	setunion(x, y)
does y belong to the set x	setsearch($x, y, \{flag\}$)
set of all $f(x, y)$, $x \in X$, $y \in Y$	setbinop(f, X, Y)
is x a set ?	setisset(x)

Lists. create empty list: $L = \text{List}()$

append x to list L	listput($L, x, \{i\}$)
remove i -th component from list L	listpop($L, \{i\}$)
insert x in list L at position i	listinsert(L, x, i)
sort the list L in place	listsort($L, \{flag\}$)

Maps. create empty dictionary: $M = \text{Map}()$

attach value v to key k	mapput(M, k, v)
recover value attach to key k or error	mapget(M, k)
is key k in the dict? (set v to $M(k)$)	mapisdefined($M, k, \{\&v\}$)
remove k from map domain	mapdelete(M, k)

GP Programming

User functions and closures

x, y are formal parameters; y defaults to Pi if parameter omitted; z, t are local variables (lexical scope), z initialized to 1.

```
fun(x, y=Pi) = my(z=1, t); seq
fun = (x, y=Pi) -> my(z=1, t); seq
```

attach help message h to s	addhelp(s, h)
undefine symbol s (also kills help)	kill(s)
Control Statements (X : formal parameter in expression seq)	
if $a \neq 0$, evaluate seq_1 , else seq_2	if($a, \{seq_1\}, \{seq_2\}$)

eval. seq for $a \leq X \leq b$	for($X = a, b, seq$)
... for $X \in v$	foreach(v, X, seq)
... for primes $a \leq X \leq b$	forprime($X = a, b, seq$)
... for primes $\equiv a \pmod q$	forprimestep($X = a, b, q, seq$)
... for composites $a \leq X \leq b$	forcomposite($X = a, b, seq$)
... for $a \leq X \leq b$ stepping s	forstep($X = a, b, s, seq$)
... for X dividing n	fordiv(n, X, seq)
... $X = [n, factor(n)]$, $a \leq n \leq b$	forfactored($X = a, b, seq$)
... as above, n squarefree	forsquarefree($X = a, b, seq$)
... $X = [d, factor(d)]$, $d n$	fordivfactored(n, X, seq)
multivariable for, lex ordering	forvec($X = v, seq$)

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loop over partitions of n
... permutations of S
... subsets of $\{1, \dots, n\}$
... k -subsets of $\{1, \dots, n\}$
... vectors $v, q(v) \leq B; q > 0$
... $H < G$ finite abelian group
evaluate seq until $a \neq 0$
while $a \neq 0$, evaluate seq
exit n innermost enclosing loops
start new iteration of n -th enclosing loop
return x from current subroutine

Exceptions, warnings

raise an exception / warning
type of error message E
try seq_1 , evaluate seq_2 on error

Functions with closure arguments / results

number of arguments of f
select from v according to f
apply f to all entries in v
evaluate $f(a_1, \dots, a_n)$
evaluate $f(\dots f(f(a_1, a_2), a_3) \dots, a_n)$
calling function as closure

Sums & Products

sum $X = a$ to $X = b$, initialized at x
sum entries of vector v
product of all vector entries
sum $expr$ over divisors of n
... assuming $expr$ multiplicative
product $a \leq X \leq b$, initialized at x
product over primes $a \leq X \leq b$

Sorting

sort x by k -th component
min. m of x ($m = x[i]$), max.
does y belong to x , sorted wrt. f
 $\prod g^x \rightarrow$ factorization (\Rightarrow sorted, unique g)

Input/Output

print with/without $\backslash n$, TeX format
pretty print matrix
print fields with separator
formatted printing
write $args$ to file
write x in binary format
read file into GP
... return as vector of lines
... return as vector of strings
read a string from keyboard

Files and file descriptors

File descriptors allow efficient small consecutive reads or writes from or to a given file. The argument n below is always a descriptor, attached to a file in **r**(ead), **w**(rite) or **a**(ppend) mode.

get descriptor n for file $path$ in given $mode$
... from shell cmd output (pipe)
close descriptor
commit pending write operations
read logical line from file
... raw line from file
write $s \backslash n$ to file
... write s to file

forpart($p = n, seq$)
forperm(S, p, seq)
forsubset(n, p, seq)
forsubset($[n, k], p, seq$)
forqfvec(v, q, b, seq)
forsubgroup($H = G$)
until(a, seq)
while(a, seq)
break($\{n\}$)
next($\{n\}$)
return($\{x\}$)

error(), warning()
errname(E)
iferr(seq_1, E, seq_2)
arity(f)
select(f, v)
apply(f, v)
call(f, a)
fold(f, a)
self()

sum($X = a, b, expr, \{x\}$)
vecsum(v)
vecprod(v)
sumdiv($n, X, expr$)
sumdivmult($n, X, expr$)
prod($X = a, b, expr, \{x\}$)
prodeuler($X = a, b, expr$)

vecsort($x, \{k\}, \{fl = 0\}$)
vecmin($x, \{\&i\}$), vecmax
vecsearch($x, y, \{f\}$)
matreduce(m)

print, print1, printtex
printp
printsep(sep, \dots), printsep1
printf()
write, write1, writetex($file, args$)
writebin($file, x$)
read($\{file\}$)
readvec($\{file\}$)
readstr($\{file\}$)
input()

fileopen($path, mode$)
fileextern(cmd)
fileclose(n)
fileflush(n)
fileread(n)
filereadstr(n)
filewrite(n, s)
filewrite1(n, s)

Timers

CPU time in ms and reset timer
CPU time in ms since gp startup
time in ms since UNIX Epoch
timeout command after s seconds

Interface with system

allocates a new stack of s bytes
alias old to new
install function from library
execute system command a
... and feed result to GP
... returning GP string
get $\$VAR$ from environment
expand env. variable in string

Parallel evaluation

These functions evaluate their arguments in parallel (pthreads or MPI); args. must not access global variables (use **export** for this) and must be free of side effects. Enabled if threading engine is not *single* in gp header.

evaluate f on $x[1], \dots, x[n]$
evaluate closures $f[1], \dots, f[n]$
as **select**
as **sum**
as **vector**
eval f for $i = a, \dots, b$
... for each element x in v
... for p prime in $[a, b]$
... for $p = a \bmod q$
... multivariate
export x to parallel world
... all dynamic variables
frees exported value x
... all exported values

Linear Algebra

dimensions of matrix x
multiply two matrices
... assuming result is diagonal
concatenation of x and y
extract components of x
transpose of vector or matrix x
adjoint of the matrix x
eigenvectors/values of matrix x
characteristic/minimal polynomial of x
trace/determinant of matrix x
permanent of matrix x
Frobenius form of x
QR decomposition
apply **matqr**'s transform to v

Constructors & Special Matrices

$\{g(x): x \in v \text{ s.t. } f(x)\}$
 $\{x: x \in v \text{ s.t. } f(x)\}$
 $\{g(x): x \in v\}$
row vec. of $expr$ eval'ed at $1 \leq i \leq n$
col. vec. of $expr$ eval'ed at $1 \leq i \leq n$
vector of small ints

gettime()
getabstime()
getwalltime()
alarm($s, expr$)
allocatemem($\{s\}$)
alias(new, old)
install($f, code, \{gpf\}, \{lib\}$)
system(a)
extern(a)
externstr(a)
getenv("VAR")
strexpend(x)

parapply(f, x)
pareval(f)
parselect($f, A, \{flag\}$)
parsum($i = a, b, expr$)
parvector($n, i, \{expr\}$)
parfor($i = a, \{b\}, f, \{r\}, \{f_2\}$)
parforeach($v, x, f, \{r\}, \{f_2\}$)
parforprime($p = a, \{b\}, f, \{r\}, \{f_2\}$)
parforprimestep($p = a, \{b\}, q, f, \{r\}, \{f_2\}$)
parforvec($X = v, f, \{r\}, \{f_2\}, \{flag\}$)
export(x)
exportall()
unexport(x)
unexportall()

matsize(x)
 $x * y$
matmultodiagonal(x, y)
concat($x, \{y\}$)
vecextract($x, y, \{z\}$)
 $x \sim$, mattranspose(x)
matadjoint(x)
mateigen(x)
charpoly(x), minpoly(x)
trace(x), matdet(x)
matpermanent(x)
matfrobenius(x)
matqr(x)
mathouseholder(Q, v)

$[g(x) \mid x \leftarrow v, f(x)]$
 $[x \mid x \leftarrow v, f(x)]$
 $[g(x) \mid x \leftarrow v]$
vector($n, \{i\}, \{expr\}$)
vectorv($n, \{i\}, \{expr\}$)
vectorsmall($n, \{i\}, \{expr\}$)

$[c, c \cdot x, \dots, c \cdot x^n]$
 $[1, 2^x, \dots, n^x]$
matrix $1 \leq i \leq m, 1 \leq j \leq n$
define matrix by blocks
diagonal matrix with diagonal x
is x diagonal?
 $x \cdot \text{matdiagonal}(d)$
 $n \times n$ identity matrix
Hessenberg form of square matrix x
 $n \times n$ Hilbert matrix $H_{ij} = (i + j - 1)^{-1}$
 $n \times n$ Pascal triangle
companion matrix to polynomial x
Sylvester matrix of x and y

Gaussian elimination

kernel of matrix x
intersection of column spaces of x and y
solve $MX = B$ (M invertible)
one sol of $M * X = B$
basis for image of matrix x
columns of x *not* in **matimage**
supplement columns of x to get basis
rows, cols to extract invertible matrix
rank of the matrix x
solve $MX = B \bmod D$
image mod D
kernel mod D
inverse mod D
determinant mod D

Lattices & Quadratic Forms

Quadratic forms

evaluate ${}^t x Q y$
evaluate ${}^t x Q x$
signature of quad form ${}^t y * x * y$
decomp into squares of ${}^t y * x * y$
eigenvalues/vectors for real symmetric x

HNF and SNF

upper triangular Hermite Normal Form
HNF of x where d is a multiple of $\det(x)$
multiple of $\det(x)$
HNF of $(x \mid \text{diagonal}(D))$
elementary divisors of x
elementary divisors of $\mathbf{Z}[a]/(f'(a))$
integer kernel of x
 \mathbf{Z} -module $\leftrightarrow \mathbf{Q}$ -vector space

Lattices

LLL-algorithm applied to columns of x
... for Gram matrix of lattice
find up to m sols of **qfnorm**(x, y) $\leq b$
 $v, v[i] :=$ number of y s.t. **qfnorm**(x, y) = i
perfection rank of x
find isomorphism between q and Q
precompute for isomorphism test with q
automorphism group of q

Based on an earlier version by Joseph H. Silverman
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convert `qfauto` for GAP/Magma `qfautoexport(G, {flag})`
 orbits of V under $G \subset GL(V)$ `qforbits(G, V)`

Polynomials & Rational Functions

all defined polynomial variables `variables()`
 get var. of highest priority (higher than v) `varhigher(name, {v})`
 ... of lowest priority (lower than v) `varlower(name, {v})`

Coefficients, variables and basic operators

degree of f `poldegree(f)`
 coef. of degree n of f , leading coef. `polcoef(f, n)`, `pollead`
 main variable / all variables in f `variable(f)`, `variables(f)`
 replace x by y in f `subst(f, x, y)`
 evaluate f replacing vars by their value `eval(f)`
 replace polynomial expr. $T(x)$ by y in f `substpol(f, T, y)`
 replace x_1, \dots, x_n by y_1, \dots, y_n in f `substvec(f, x, y)`

$f \in A[x]$; reciprocal polynomial $x^{\deg f} f\left(\frac{1}{x}\right)$ `polrecip(f)`
 gcd of coefficients of f `content(f)`
 derivative of f w.r.t. x `deriv(f, {x})`
 ... n -th derivative of f `derivn(f, n, {x})`
 formal integral of f w.r.t. x `intformal(f, {x})`
 formal sum of f w.r.t. x `sumformal(f, {x})`

Constructors & Special Polynomials

interpolation polynomial at $(x[1], y[1]), \dots, (x[n], y[n])$, evaluated at t , with error estimate e `polinterpolate(x, {y}, {t}, {&e})`
 $T_n/U_n, H_n$ `polchebyshev(n)`, `polhermite(n)`
 $P_n, L_n^{(\alpha)}$ `pollegendre(n)`, `pollaguerre(n, a)`
 n -th cyclotomic polynomial Φ_n `polcyclo(n)`
 return n if $f = \Phi_n$, else 0 `poliscyclo(f)`
 is f a product of cyclotomic polynomials? `poliscycloprod(f)`
 Zagier's polynomial of index (n, m) `polzagier(n, m)`

Resultant, elimination

discriminant of polynomial f `poldisc(f)`
 find factors of `poldisc(f)` `poldiscfactors(f)`
 resultant $R = \text{Res}_v(f, g)$ `polresultant(f, g, {v})`
 $[u, v, R]$, $xu + yv = \text{Res}_v(f, g)$ `polresultantext(x, y, {v})`
 solve Thue equation $f(x, y) = a$ `thue(t, a, {sol})`
 initialize t for Thue equation solver `thueinit(f)`

Roots and Factorization (Complex/Real)

complex roots of f `polroots(f)`
 bound complex roots of f `polrootsbound(f)`
 number of real roots of f (in $[a, b]$) `polsturm(f, {[a, b]})`
 real roots of f (in $[a, b]$) `polrootsreal(f, {[a, b]})`
 complex embeddings of `t_POLMOD z` `conjvec(z)`

Roots and Factorization (Finite fields)

factor f mod p , roots `factormod(f, p)`, `polrootsmod`
 factor f over $\mathbf{F}_p[x]/(T)$, roots `factormod(f, [T, p])`, `polrootsmod`
 squarefree factorization of f in $\mathbf{F}_q[x]$ `factormodSQF(f, {D})`
 distinct degree factorization of f in $\mathbf{F}_q[x]$ `factormodDDF(f, {D})`

Roots and Factorization (p -adic fields)

factor f over \mathbf{Q}_p , roots `factorpadic(f, p, r)`, `polrootspadic`
 p -adic root of f congruent to a mod p `padicappr(f, a)`
 Newton polygon of f for prime p `newtonpoly(f, p)`
 Hensel lift $A/\text{lc}(A) = \prod_i B[i] \pmod{p^e}$ `polhensellift(A, B, p, e)`
 $T = \prod (x - z_i) \mapsto \prod (x - \omega(z_i)) \in \mathbf{Z}_p[x]$ `polteichmuller(T, p, e)`
 extensions of \mathbf{Q}_p of degree N `padicfields(p, N)`

Pari-GP reference card

(PARI-GP version 2.13.3)

Roots and Factorization (Miscellaneous)

symmetric powers of roots of f up to n `polysym(f, n)`
 Graeffe transform of f , $g(x^2) = f(x)f(-x)$ `polgraeffe(f)`
 factor f over coefficient field `factor(f)`
 cyclotomic factors of $f \in \mathbf{Q}[X]$ `polcyclofactors(f)`

Finite Fields

A finite field is encoded by any element (`t_FFELT`).
 find irreducible $T \in \mathbf{F}_p[x]$, $\deg T = n$ `ffinit(p, n, {x})`
 Create t in $\mathbf{F}_q \simeq \mathbf{F}_p[t]/(T)$ `t = ffgen(T, 't)`
 ... indirectly, with implicit T `t = ffgen(q, 't); T = t.mod`
 map m from $\mathbf{F}_q \ni a$ to $\mathbf{F}_{q^k} \ni b$ `m = ffbend(a, b)`
 build $K = \mathbf{F}_q[x]/(P)$ extending $\mathbf{F}_q \ni a$, `ffextend(a, P)`
 evaluate map m on x `ffmap(m, x)`
 inverse map of m `ffinvm(m)`
 compose maps $m \circ n$ `ffcompom(m, n)`
 x as polmod over codomain of map m `ffmaprel(m, x)`
 F^n over $\mathbf{F}_q \ni a$ `fffrobenius(a, n)`
 $\#\{\text{monic irred. } T \in \mathbf{F}_q[x], \deg T = n\}$ `ffnbirred(q, n)`

Formal & p -adic Series

truncate power series or p -adic number `truncate(x)`
 valuation of x at p `valuation(x, p)`
Dirichlet and Power Series
 Taylor expansion around 0 of f w.r.t. x `taylor(f, x)`
 Laurent series of closure F up to x^k `laurentseries(f, k)`
 $\sum a_k b_k t^k$ from $\sum a_k t^k$ and $\sum b_k t^k$ `serconvol(a, b)`
 $f = \sum a_k t^k$ from $\sum (a_k/k!) t^k$ `serlaplace(f)`
 reverse power series F so $F(f(x)) = x$ `serreverse(f)`
 remove terms of degree $< n$ in f `serchop(f, n)`
 Dirichlet series multiplication / division `dirmul, dirdiv(x, y)`
 Dirichlet Euler product (b terms) `direuler(p = a, b, expr)`

Transcendental and p -adic Functions

real, imaginary part of x `real(x)`, `imag(x)`
 absolute value, argument of x `abs(x)`, `arg(x)`
 square/ n th root of x `sqrt(x)`, `sqrtn(x, n, {&z})`
 all n -th roots of 1 `rootsof1(n)`
 FFT of $[f_0, \dots, f_{n-1}]$ `w = fftinit(n)`, `fft/fftinw(w, f)`
 trig functions `sin, cos, tan, cotan, sinc`
 inverse trig functions `asin, acos, atan`
 hyperbolic functions `sinh, cosh, tanh, cotanh`
 inverse hyperbolic functions `asinh, acosh, atanh`
 $\log(x)$, $\log(1+x)$, e^x , $e^x - 1$ `log, loglp, exp, expm1`
 Euler Γ function, $\log \Gamma$, Γ'/Γ `gamma, lngamma, psi`
 half-integer gamma function $\Gamma(n+1/2)$ `gammah(n)`
 Riemann's zeta $\zeta(s) = \sum n^{-s}$ `zeta(s)`
 $\sum_{n \leq N} n^s$ `dirpowerssum(N, s)`
 Hurwitz's $\zeta(s, x) = \sum (n+x)^{-s}$ `zetahurwitz(s, x)`
 multiple zeta value (MZV), $\zeta(s_1, \dots, s_k)$ `zetamult(s, {T})`
 all MZVs for weight $\sum s_i = n$ `zetamultall(n)`
 convert MZV id to $[s_1, \dots, s_k]$ `zetamultconvert(f, {flag})`
 MZV dual sequence `zetamultdual(s)`
 multiple polylog $Li_{s_1, \dots, s_k}(z_1, \dots, z_k)$ `polylogmult(s, z)`

incomplete Γ function ($y = \Gamma(s)$) `incgam(s, x, {y})`
 complementary incomplete Γ `incgamc(s, x)`
 $\int_x^\infty e^{-t} dt/t$, $(2/\sqrt{\pi}) \int_x^\infty e^{-t^2} dt$ `eint1, erfc`
 elliptic integral of 1st and 2nd kind `ellK(k)`, `ellE(k)`
 dilogarithm of x `dilog(x)`
 m -th polylogarithm of x `polylog(m, x, {flag})`
 U -confluent hypergeometric function `hyperu(a, b, u)`
 Hypergeometric ${}_pF_q(A, B; z)$ `hypergeom(A, B, z)`
 Bessel $J_n(x)$, $J_{n+1/2}(x)$ `besselj(n, x)`, `besseljh(n, x)`
 Bessel I_ν , K_ν , H_ν^1 , H_ν^2 , Y_ν `(bessel)i, k, h1, h2, y`
 Airy functions $A_i(x)$, $B_i(x)$ `airy(x)`
 Lambert W : x s.t. $xe^x = y$ `lambertw(y)`
 Teichmuller character of p -adic x `teichmuller(x)`

Iterations, Sums & Products

Numerical integration for meromorphic functions

Behaviour at endpoint for Double Exponential (DE) methods: either a scalar ($a \in \mathbf{C}$, regular) or $\pm\infty$ (decreasing at least as x^{-2}) or $(x-a)^{-\alpha}$ singularity `[a, a]`
 exponential decrease $e^{-\alpha|x|}$ `[±∞, a], α > 0`
 slow decrease $|x|^\alpha$ `... α < -1`
 oscillating as $\cos(kx)$ `α = kI, k > 0`
 oscillating as $\sin(kx)$ `α = -kI, k > 0`

numerical integration `intnum(x = a, b, f, {T})`
 weights T for `intnum` `intnuminit(a, b, {m})`
 weights T incl. kernel K `intfuncinit(t = a, b, K, {m})`
 integrate $(2i\pi)^{-1} f$ on circle $|z - a| = R$ `intcirc(x = a, R, f, {T})`

Other integration methods

n -point Gauss-Legendre `intnumgauss(x = a, b, f, {n})`
 weights for n -point Gauss-Legendre `intnumgaussinit({n})`
 Romberg integration (low accuracy) `intnumromb(x = a, b, f, {flag})`

Numerical summation

sum of series $f(n)$, $n \geq a$ (low accuracy) `suminf(n = a, expr)`
 sum of alternating/positive series `sumalt, sumpos`
 sum of series using Euler-Maclaurin `sumnum(n = a, f, {T})`
 $\sum_{n \geq a} F(n)$, F rational function `sumnumrat(F, a)`
 $\dots \sum_{p \geq a} F(p^s)$ `sumeulerrat(F, {s = 1}, {a = 2})`
 weights for `sumnum`, a as in DE `sumnuminit({∞, a})`
 sum of series by Monien summation `sumnummonien(n = a, f, {T})`
 weights for `sumnummonien` `sumnummonieninit({∞, a})`
 sum of series using Abel-Plana `sumnumap(n = a, f, {T})`
 weights for `sumnumap`, a as in DE `sumnumapinit({∞, a})`
 sum of series using Lagrange `sumnumlagrange(n = a, f, {T})`
 weights for `sumnumlagrange` `sumnumlagrangeinit`

Products

product $a \leq X \leq b$, initialized at x `prod(X = a, b, expr, {x})`
 product over primes $a \leq X \leq b$ `prodeuler(X = a, b, expr)`
 infinite product $a \leq X \leq \infty$ `prodinf(X = a, expr)`
 $\prod_{n \geq a} F(n)$, F rational function `prodnrat(F, a)`
 $\prod_{p \geq a} F(p^s)$ `prodeulerrat(F, {s = 1}, {a = 2})`

Other numerical methods

real root of f in $[a, b]$; bracketed root	<code>solve($X = a, b, f$)</code>
... interval splitting, step s	<code>solvestep($X = a, b, s, f, \{flag = 0\}$)</code>
limit of $f(t)$, $t \rightarrow \infty$	<code>limitnum($f, \{\alpha\}$)</code>
asymptotic expansion of f (rational)	<code>asymnum($f, \{\alpha\}$)</code>
... $N + 1$ terms as floats	<code>asymnumraw($f, N, \{\alpha\}$)</code>
numerical derivation w.r.t x : $f'(a)$	<code>derivnum($x = a, f$)</code>
evaluate continued fraction F at t	<code>contfraceval($F, t, \{L\}$)</code>
power series to cont. fraction (L terms)	<code>contfracinit($S, \{L\}$)</code>
Padé approximant (deg. denom. $\leq B$)	<code>bestapprPade($S, \{B\}$)</code>

Elementary Arithmetic Functions

vector of binary digits of $ x $	<code>binary(x)</code>
bit number n of integer x	<code>bittest(x, n)</code>
Hamming weight of integer x	<code>hammingweight(x)</code>
digits of integer x in base B	<code>digits($x, \{B = 10\}$)</code>
sum of digits of integer x in base B	<code>sumdigits($x, \{B = 10\}$)</code>
integer from digits	<code>fromdigits($v, \{B = 10\}$)</code>
ceiling/floor/fractional part	<code>ceil, floor, frac</code>
round x to nearest integer	<code>round($x, \{\&e\}$)</code>
truncate x	<code>truncate($x, \{\&e\}$)</code>
gcd/LCM of x and y	<code>gcd(x, y), lcm(x, y)</code>
gcd of entries of a vector/matrix	<code>content(x)</code>

Primes and Factorization

extra prime table	<code>addprimes()</code>
add primes in v to prime table	<code>addprimes(v)</code>
remove primes from prime table	<code>removeprimes(v)</code>
Chebyshev $\pi(x)$, n -th prime p_n	<code>primepi(x), prime(n)</code>
vector of first n primes	<code>primes(n)</code>
smallest prime $\geq x$	<code>nextprime(x)</code>
largest prime $\leq x$	<code>preprime(x)</code>
factorization of x	<code>factor($x, \{lim\}$)</code>
... selecting specific algorithms	<code>factorint($x, \{flag = 0\}$)</code>
$n = df^2$, d squarefree/fundamental	<code>core($n, \{fl\}$), coredisc</code>
certificate for (prime) N	<code>primecert(N)</code>
verifies a certificate c	<code>primecertisvalid(c)</code>
convert certificate to Magma/PRIMO	<code>primecertexport</code>
recover x from its factorization	<code>factorback($f, \{e\}$)</code>
$x \in \mathbf{Z}$, $ x \leq X$, $\gcd(N, P(x)) \geq N$	<code>zncoppersmith($P, N, X, \{B\}$)</code>
divisors of N in residue class r mod s	<code>divisorslenstra(N, r, s)</code>

Divisors and multiplicative functions

number of prime divisors $\omega(n)$ / $\Omega(n)$	<code>omega(n), bigomega</code>
divisors of n / number of divisors $\tau(n)$	<code>divisors(n), numdiv</code>
sum of (k -th powers of) divisors of n	<code>sigma($n, \{k\}$)</code>
Möbius μ -function	<code>moebius(x)</code>
Ramanujan's τ -function	<code>ramanujantau(x)</code>

Combinatorics

factorial of x	<code>x!</code> or <code>factorial(x)</code>
binomial coefficient $\binom{x}{k}$	<code>binomial($x, \{k\}$)</code>
Bernoulli number B_n as real/rational	<code>bernreal(n), bernfrac</code>
$[B_0, B_2, \dots, B_{2k}]$	<code>bernvec(k)</code>
Bernoulli polynomial $B_n(x)$	<code>bernpol($n, \{x\}$)</code>
Euler numbers	<code>eulerfrac, eulervec</code>
Euler polynomials $E_n(x)$	<code>eulerpol($n, \{x\}$)</code>
Eulerian polynomials $A_n(x)$	<code>eulerianpol</code>
Fibonacci number F_n	<code>fibonacci(n)</code>
Stirling numbers $s(n, k)$ and $S(n, k)$	<code>stirling($n, k, \{flag\}$)</code>

Pari-GP reference card

(PARI-GP version 2.13.3)

number of partitions of n	<code>numbpart(n)</code>
k -th permutation on n letters	<code>numtoperm(n, k)</code>
... index k of permutation v	<code>permtotnum(v)</code>
order of permutation p	<code>permorder(p)</code>
signature of permutation p	<code>permsign(p)</code>
cyclic decomposition of permutation p	<code>permcycles(p)</code>
Multiplicative groups ($\mathbf{Z}/N\mathbf{Z}$)*, \mathbf{F}_q^*	
Euler ϕ -function	<code>eulerphi(x)</code>
multiplicative order of x (divides ϕ)	<code>znorder($x, \{o\}$), fforder</code>
primitive root mod q / x .mod	<code>znprimroot(q), fprimroot(x)</code>
structure of ($\mathbf{Z}/n\mathbf{Z}$)*	<code>znstar(n)</code>
discrete logarithm of x in base g	<code>znlog($x, g, \{o\}$), fflag</code>
Kronecker-Legendre symbol $(\frac{x}{y})$	<code>kronecker(x, y)</code>
quadratic Hilbert symbol (at p)	<code>hilbert($x, y, \{p\}$)</code>
Euclidean algorithm, continued fractions	
CRT: solve $z \equiv x$ and $z \equiv y$	<code>chinese(x, y)</code>
minimal u, v so $ux + yv = \gcd(x, y)$	<code>gcdext(x, y)</code>
half-gcd algorithm	<code>halfgcd(x, y)</code>
continued fraction of x	<code>contfrac($x, \{b\}, \{lmax\}$)</code>
last convergent of continued fraction x	<code>contfracpnqn(x)</code>
rational approximation to x (den. $\leq B$)	<code>bestappr($x, \{B\}$)</code>
recognize $x \in \mathbf{C}$ as polmod mod $T \in \mathbf{Z}[X]$	<code>bestapprnf(x, T)</code>

Miscellaneous

integer square / n -th root of x	<code>sqrtnint(x, n)</code>
largest integer e s.t. $b^e \leq x$, $e = \lfloor \log_b(x) \rfloor$	<code>logint($x, b, \{\&z\}$)</code>

Characters

Let $cyc = [d_1, \dots, d_k]$ represent an abelian group $G = \oplus (\mathbf{Z}/d_j\mathbf{Z}) \cdot g_j$ or any structure G affording a `.cyc` method; e.g. `znstar($q, 1$)` for Dirichlet characters. A character χ is coded by $[c_1, \dots, c_k]$ such that $\chi(g_j) = e(n_j/d_j)$.
 $\chi \cdot \psi$; χ^{-1} ; $\chi \cdot \psi^{-1}$; χ^k `charm, charconj, chardiv, charpow`
order of χ `charorder(cyc, χ)`
kernel of χ `charker(cyc, χ)`
 $\chi(x)$, G a GP group structure `chareval($G, \chi, x, \{z\}$)`
Galois orbits of characters `chargalois(G)`

Dirichlet Characters

initialize $G = (\mathbf{Z}/q\mathbf{Z})^*$ `G = znstar($q, 1$)`
convert datum D to $[G, \chi]$ `znchar(D)`
is χ odd? `zncharisodd(G, χ)`
real $\chi \rightarrow$ Kronecker symbol (D/\cdot) `znchartokronecker(G, χ)`
conductor of χ `zncharconductor(G, χ)`
 $[G_0, \chi_0]$ primitive attached to χ `znchartoprimitive(G, χ)`
induce $\chi \in \hat{G}$ to $\mathbf{Z}/N\mathbf{Z}$ `zncharinduce(G, χ, N)`
 χp `znchardecompose(G, χ, p)`
 $\prod_p (Q, N) \chi p$ `znchardecompose(G, χ, Q)`
complex Gauss sum $G_a(\chi)$ `znchargauss(G, χ)`

Conrey labelling

Conrey label $m \in (\mathbf{Z}/q\mathbf{Z})^* \rightarrow$ character `znconreychar(G, m)`
character \rightarrow Conrey label `znconreyexp(G, χ)`
log on Conrey generators `znconreylog(G, m)`
conductor of χ (χ_0 primitive) `znconreyconductor($G, \chi, \{\chi_0\}$)`

True-False Tests

is x the disc. of a quadratic field?	<code>isfundamental(x)</code>
is x a prime?	<code>isprime(x)</code>
is x a strong pseudo-prime?	<code>ispseudoprime(x)</code>
is x square-free?	<code>issquarefree(x)</code>
is x a square?	<code>issquare($x, \{\&n\}$)</code>
is x a perfect power?	<code>ispower($x, \{k\}, \{\&n\}$)</code>
is x a perfect power of a prime? ($x = p^n$)	<code>isprimepower($x, \&n$)</code>
... of a pseudoprime?	<code>ispseudoprimepower($x, \&n$)</code>
is x powerful?	<code>ispowerful(x)</code>
is x a totient? ($x = \varphi(n)$)	<code>istotient($x, \{\&n\}$)</code>
is x a polygonal number? ($x = P(s, n)$)	<code>ispolygonal($x, s, \{\&n\}$)</code>
is pol irreducible?	<code>polisirreducible(pol)</code>

Graphic Functions

crude graph of $expr$ between a and b	<code>plot($X = a, b, expr$)</code>
High-resolution plot (immediate plot)	
plot $expr$ between a and b	<code>plotoh($X = a, b, expr, \{flag\}, \{n\}$)</code>
plot points given by lists lx, ly	<code>plotthraw($lx, ly, \{flag\}$)</code>
terminal dimensions	<code>plotsizes()</code>
Rectwindow functions	
init window w , with size x, y	<code>plotinit(w, x, y)</code>
erase window w	<code>plotkill(w)</code>
copy w to w_2 with offset (dx, dy)	<code>plotcopy(w, w_2, dx, dy)</code>
clips contents of w	<code>plotclip(w)</code>
scale coordinates in w	<code>plotscale(w, x_1, x_2, y_1, y_2)</code>
<code>plotoh</code> in w	<code>plotrecth($w, X = a, b, expr, \{flag\}, \{n\}$)</code>
<code>plotthraw</code> in w	<code>plotrecthraw($w, data, \{flag\}$)</code>
draw window w_1 at $(x_1, y_1), \dots$	<code>plotdraw([[w_1, x_1, y_1], ...])</code>

Low-level Rectwindow Functions

set current drawing color in w to c	<code>plotcolor(w, c)</code>
current position of cursor in w	<code>plotcursor(w)</code>
write s at cursor's position	<code>plotstring(w, s)</code>
move cursor to (x, y)	<code>plotmove(w, x, y)</code>
move cursor to $(x + dx, y + dy)$	<code>plotrmove(w, dx, dy)</code>
draw a box to (x_2, y_2)	<code>plotbox(w, x_2, y_2)</code>
draw a box to $(x + dx, y + dy)$	<code>plotrbox(w, dx, dy)</code>
draw polygon	<code>plotlines($w, lx, ly, \{flag\}$)</code>
draw points	<code>plotpoints(w, lx, ly)</code>
draw line to $(x + dx, y + dy)$	<code>plotrline(w, dx, dy)</code>
draw point $(x + dx, y + dy)$	<code>plotrpoint(w, dx, dy)</code>

Convert to Postscript or Scalable Vector Graphics

The format f is either "ps" or "svg".
as `plotoh` `plotexport($f, X = a, b, expr, \{flag\}, \{n\}$)`
as `plotthraw` `plotthrawexport($f, lx, ly, \{flag\}$)`
as `plotdraw` `plotexport($f, [[w_1, x_1, y_1], ...])$`

Based on an earlier version by Joseph H. Silverman
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