# Étale cohomology of surfaces, degeneracy of Galois representations, and a conjectural geometric explanation for ramification

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# Goal

Let $C/\mathbb{Q}$ be a "nice" curve of genus $g$.
Let $J$ be its Jacobian, and fix a prime $\ell \in \mathbb{N}$.
$\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ acts on $J(\overline{\mathbb{Q}})[\ell] \rightsquigarrow$ Galois representation

$$\rho_{C,\ell} : \mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q}) \longrightarrow \mathrm{GSp}_{2g}(\mathbb{F}_\ell).$$

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Motivation(s):

- Point counting:

$$\begin{aligned} \rho(\text{Frob}_p) &\rightsquigarrow \#X(\mathbb{F}_p) \bmod \ell \\ &\underset{\text{CRT}}{\rightsquigarrow} \text{Get } \#X(\mathbb{F}_p) \text{ in } (\log p)^{O(1)}. \end{aligned}$$

- Modular curves $\rightsquigarrow$ Galois representations attached to modular forms $\rightsquigarrow$ Get $a_p(f)$ in $(\log p)^{O(1)}$.
- Interesting number-theoretic objects, e.g. polynomials with Galois group $\subseteq \text{GSp}_{2g}(\mathbb{F}_\ell)$ and with controlled ramification.

# Division polynomials

Suppose $\alpha \in \mathbb{Q}(J)$ is defined and injective on $J[\ell]$.
$\rightsquigarrow$ "Division polynomial"

$$R_{C,\ell}(x) = \prod_{t \in J[\ell]} \big(x - \alpha(t)\big) \in \mathbb{Q}[x].$$

Splitting field $= \mathbb{Q}(t | t \in J[\ell])$.
Galois group $= \operatorname{Im} \rho_{C,\ell} \leqslant \mathrm{GSp}_{2g}(\mathbb{F}_\ell)$.

**Theorem (Néron-Ogg-Shafarevic)**

*Unramified away from $\ell$ and primes of bad reduction of $C$.*

# $p$-adic algorithm to compute $\rho_{C,\ell}$

Fix a small-ish prime $p \neq \ell$ of good reduction of $C$.

1. Determine $L_p(x) = \det\left(x - \mathrm{Frob}_p\,|_J\right) \in \mathbb{Z}[x]$.
2. Find $q = p^a$ such that $J(\overline{\mathbb{F}_p})[\ell] \subseteq J(\mathbb{F}_q)$.
3. Compute $N = \#J(\mathbb{F}_q) = \mathrm{Res}(L_p(x), x^a - 1)$.
   Write $N = \ell^v N'$, $\ell \nmid N'$.
4. Take random $x \in J(\mathbb{F}_q)$
   $\rightsquigarrow N'x \in J(\mathbb{F}_q)[\ell^\infty]$
   $\rightsquigarrow \ell^{w_x} N'x \in J(\mathbb{F}_q)[\ell]$ for some $w_x \leqslant v$.
   Repeat until we get an $\mathbb{F}_\ell$-basis of $J(\mathbb{F}_q)[\ell]$.
5. Lift these points from $J(\mathbb{F}_q)[\ell]$ to $J(\mathbb{Z}_q/p^e)[\ell]$, $e \gg 1$.
6. Form all linear combinations to get all the points of $J(\mathbb{Z}_q/p^e)[\ell]$.
7. Identify $R_{C,\ell}(x) = \prod_{t \in J[\ell]} \left(x - \alpha(t)\right)$ in $\mathbb{Q}[x]$.

# Example: 2-torsion of a plane quartic

Take $C : x^3y + y^3 + y^2 - x^2 - x = 0$, investigate $J[2]$.

```
R=CrvGalRep(x^3*y+y^3+y^2-x^2-x,2,10,30)
polisirreducible(R[1])
factor(nfdisc(R[1]))
```

We get full image $\mathrm{GSp}_6(\mathbb{F}_2)$, and ramification at 2, 71, 367.

# Representations from higher étale cohomology

So we can compute with $J[\ell] \approx \mathrm{H}^1_{\text{ét}}(\text{Curve}, \mathbb{Z}/\ell\mathbb{Z})$.

Next natural step: $\mathrm{H}^2_{\text{ét}}(\text{Surface}, \mathbb{Z}/\ell\mathbb{Z})$.

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Solution: *dévissage* by Leray's spectral sequence

$$\text{“}\mathrm{H}^p(\mathrm{H}^q) \Rightarrow \mathrm{H}^{p+q}\text{”}.$$

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**Theorem (M., 2019)**

Let $S/\mathbb{Q}$ be a regular surface. For every $\rho \subset \mathrm{H}^2_{\text{ét}}(S, \mathbb{Z}/\ell\mathbb{Z})$, one can construct a curve $C/\mathbb{Q}$ such that $\rho \subset \mathrm{Jac}(C)[\ell]$ (up to twist by the cyclotomic character).

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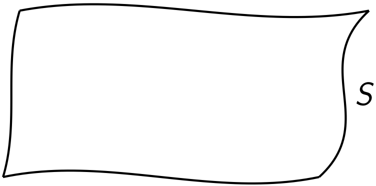

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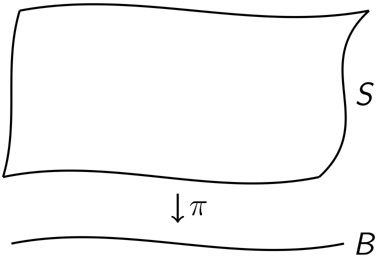

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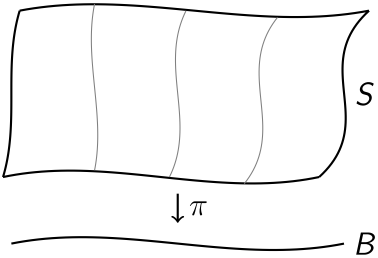

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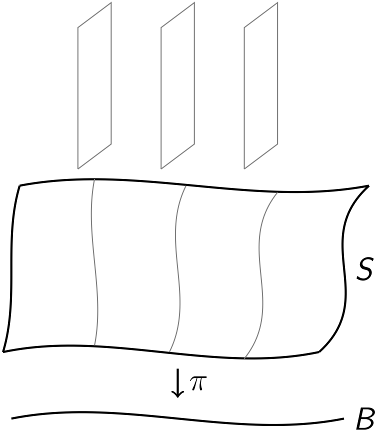

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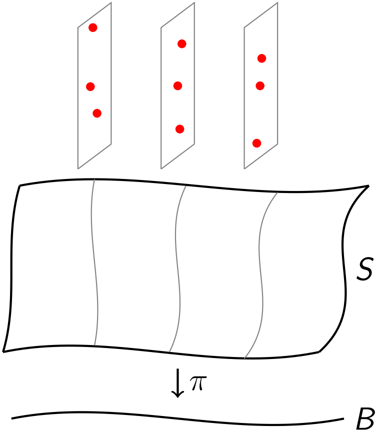

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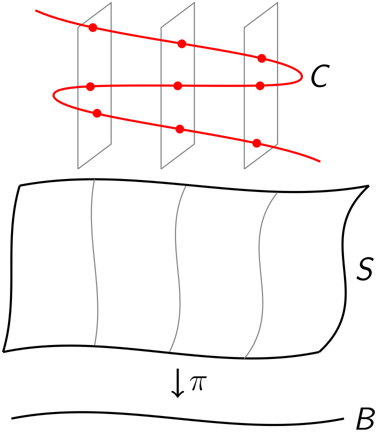

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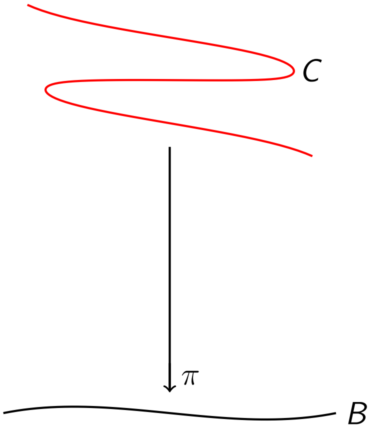


# Strategy

We have sketched a $p$-adic algorithm
Curve $C/\mathbb{Q} \rightsquigarrow \ell$-division polynomial $R_{C,\ell}(x) \in \mathbb{Q}[x]$.

This can be generalised to
Curve $C/\mathbb{Q}(t) \rightsquigarrow \ell$-division polynomial $R_{C,\ell}(x,t) \in \mathbb{Q}(t)[x]$,
by lifting $\ell$-torsion points $(p,t)$-adically.

Now suppose we want to compute $\rho \subset \mathrm{H}^2_{\text{ét}}(S, \mathbb{Z}/\ell\mathbb{Z})$.

1. Pick $\pi : S \longrightarrow B$, where $B = \mathbb{P}^1_{\mathbb{Q}}$ with coordinate $t$
   $\rightsquigarrow$ view $S$ as a curve $\mathcal{S}$ over $\mathbb{Q}(t)$.
2. Compute $\ell$-division polynomial $R_{\mathcal{S},\ell}(x,t) \in \mathbb{Q}(t)[x]$.
   $\rightsquigarrow C : R_{\mathcal{S},\ell}(x,t) = 0$ contains $\rho$ in its Jacobian.
3. Isolate $\rho$ in $T \subset \mathrm{Jac}(C)[\ell]$ by characterising $T$ by the action of $\mathrm{Frob}_p$ for a well-chosen $p$.

# Plane algebraic curves

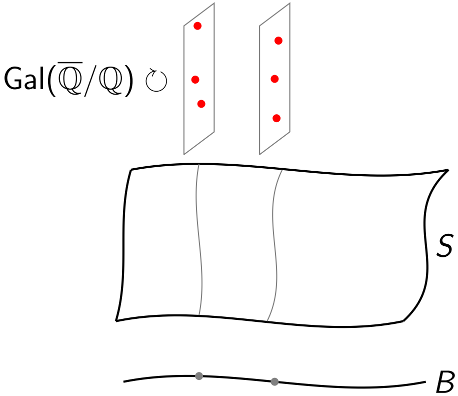
All this requires handling singular plane models $f(x, y) = 0$ of curves over $\mathbb{Q}$ or $\mathbb{Q}(t)$.

Personal PARI/GP implementation, relies on Puiseux expansions ($\approx$ factorisation in $\mathbb{Q}((x))[y]$) to resolve singularities.

```
C=CrvInit(y^12-x^4*(x-1)^9);
CrvPrint(C)
CrvHyperell(C)
CanProj(C)
```

# Families of Galois representations

The division polynomial $R_{\mathcal{S},\ell}(x,t) \in \mathbb{Q}(t)[x]$ defines a family of Galois representations parametrised by $t \in \mathbb{P}^1_{\mathbb{Q}}$.
For good fibres $t_0 \in \mathbb{P}^1_{\mathbb{Q}}$, $R_{\mathcal{S},\ell}(x, t = t_0)$ describes $\mathrm{Jac}(S_{t_0})[\ell]$




$\rightsquigarrow$ Galois group $\leqslant \mathrm{GSp}_{2g}(\mathbb{F}_\ell)$, and ramification restricted to $\ell$ and primes of bad reduction of $C_{t_0}$ by Néron-Ogg-Shafarevich.

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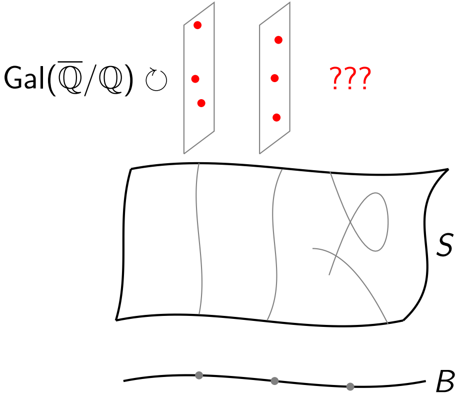


But what happens at bad fibres?

# Understanding / computing degeneracy

We would like to predict geometrically the ramification of the degenerate Galois representation, but cannot invoke usual Néron-Ogg-Shafarevic, because we are now working over a 2-dimensional base (1 geometric + 1 arithmetic).

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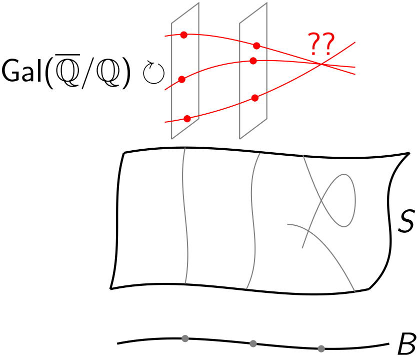
Degenerate representation: cannot simply set $t = t_0$ in $R_{\mathcal{S},\ell}(x, t)$; instead, resolve singularity by factoring over $\mathbb{Q}((t - t_0))$ .



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Degenerate representation: cannot simply set $t = t_0$ in $R_{\mathcal{S},\ell}(x,t)$; instead, resolve singularity by factoring over $\mathbb{Q}((t-t_0))$ .

Similarly, to understand the bad fibre, we do not simply take $\pi^{-1}(t_0)$; instead we consider the minimal regular model of $S/B$.

# Two families of curves

$$\mathcal{H} : y^2 = x^6 - x^4 + (t-1)(x^2+x), \quad \ell = 3.$$

| $t$ | Place decomposition | Ramification |
|---|---|---|
| $1$ | $\mathbb{Q}(\sqrt{3})^1 \cdot \mathbb{Q}(\sqrt{-1})^3 \cdot \left(\mathbb{Q}(\zeta_9)^+(\sqrt{-1})\right)^9 \cdot \left(\mathbb{Q}(\zeta_{36})^+\right)^3$ | $2, 3$ |
| $-1$ | $\mathbb{Q}(\sqrt{-21})^1 \cdot K_6^1 \cdot K_{18}^1 \cdot {K'_{18}}^3$ | $2, 3, 7, 11$ |
| $\frac{283}{256}$ | $\mathbb{Q}(\sqrt{-14})^1 \cdot {K''_{18}}^3 \cdot K_{24}^1$ | $2, 3, 7, 11$ |
| $\infty$ | $\mathbb{Q}^2 \cdot \mathbb{Q}^6 \cdot \mathbb{Q}(\sqrt{3})^4 \cdot \mathbb{Q}(\sqrt[4]{12})^4 \cdot \mathbb{Q}(\sqrt[4]{12})^{12}$ | $2, 3$ |

$$\mathcal{Q} : x^4 + (2-t)y^4 + 2x^3 + x(x+y) + (t-1)(y+x^2+x) = 0, \quad \ell = 2.$$

| $t$ | Place decomposition | Ramification |
|---|---|---|
| $1$ | $\mathbb{Q}^1 \cdot \mathbb{Q}^1 \cdot \mathbb{Q}^1 \cdot K_8^1 \cdot K_8^1 \cdot {K'_8}^2 \cdot {K''_8}^2 \cdot K_{12}^1$ | $2, 229$ |
| $2$ | $\mathbb{Q}^1 \cdot \mathbb{Q}^2 \cdot \mathbb{Q}^4 \cdot \mathbb{Q}^8 \cdot \mathbb{Q}^8 \cdot \mathbb{Q}(\sqrt{2})^4 \cdot \mathbb{Q}(\sqrt{2}, \sqrt{15})^8$ | $2, 3, 5$ |
| $\infty$ | $\mathbb{Q}^1 \cdot \mathbb{Q}^2 \cdot \mathbb{Q}^4 \cdot K_3^2 \cdot K_3^4 \cdot K_6^1 \cdot {K'''_8}^4$ | $2, 23$ |

# Two families of curves

$$\mathcal{H} : y^2 = x^6 - x^4 + (t-1)(x^2+x), \quad \ell = 3.$$

| $t$ | Place decomposition | Ramification |
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| $1$ | $\mathbb{Q}(\sqrt{3})^1 \cdot \mathbb{Q}(\sqrt{-1})^3 \cdot \left(\mathbb{Q}(\zeta_9)^+(\sqrt{-1})\right)^9 \cdot \left(\mathbb{Q}(\zeta_{36})^+\right)^3$ | $2, 3$ |
| $-1$ | $\mathbb{Q}(\sqrt{-21})^1 \cdot K_6^1 \cdot K_{18}^1 \cdot {K'_{18}}^3$ | $2, 3, 7, 11$ |
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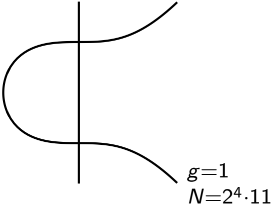
| $\infty$ | $\mathbb{Q}^2 \cdot \mathbb{Q}^6 \cdot \mathbb{Q}(\sqrt{3})^4 \cdot \mathbb{Q}(\sqrt[4]{12})^4 \cdot \mathbb{Q}(\sqrt[4]{12})^{12}$ | $2, 3$ |

Special fibre at $t = -1$ of minimal regular model of $\mathcal{H}$ in characteristic $\notin \{2, 7, 11\}$:



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$$\mathcal{H} : y^2 = x^6 - x^4 + (t-1)(x^2+x), \quad \ell = 3.$$

| $t$ | Place decomposition | Ramification |
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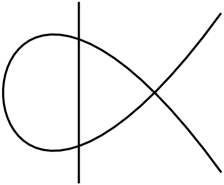
| $1$ | $\mathbb{Q}(\sqrt{3})^1 \cdot \mathbb{Q}(\sqrt{-1})^3 \cdot \left(\mathbb{Q}(\zeta_9)^+(\sqrt{-1})\right)^9 \cdot \left(\mathbb{Q}(\zeta_{36})^+\right)^3$ | $2, 3$ |
| $-1$ | $\mathbb{Q}(\sqrt{-21})^1 \cdot K_6^1 \cdot K_{18}^1 \cdot {K'_{18}}^3$ | $2, 3, 7, 11$ |
| $\frac{283}{256}$ | $\mathbb{Q}(\sqrt{-14})^1 \cdot {K''_{18}}^3 \cdot K_{24}^1$ | $2, 3, 7, 11$ |
| $\infty$ | $\mathbb{Q}^2 \cdot \mathbb{Q}^6 \cdot \mathbb{Q}(\sqrt{3})^4 \cdot \mathbb{Q}(\sqrt[4]{12})^4 \cdot \mathbb{Q}(\sqrt[4]{12})^{12}$ | $2, 3$ |

Special fibre at $t = -1$ of minimal regular model of $\mathcal{H}$ in characteristic 11:



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| $t$ | Place decomposition | Ramification |
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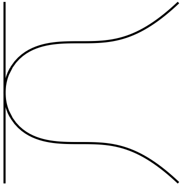
Special fibre at $t = -1$ of minimal regular model of $\mathcal{H}$ in characteristic 7:



# Two families of curves

$$\mathcal{Q} : x^4+(2-t)y^4+2x^3+x(x+y)+(t-1)(y+x^2+x) = 0, \quad \ell = 2.$$

| $t$ | Place decomposition | Ramification |
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| 1 | $\mathbb{Q}^1 \cdot \mathbb{Q}^1 \cdot \mathbb{Q}^1 \cdot K_8^1 \cdot K_8^1 \cdot K_8'^2 \cdot K_8''^2 \cdot K_{12}^1$ | $2, 229$ |
| 2 | $\mathbb{Q}^1 \cdot \mathbb{Q}^2 \cdot \mathbb{Q}^4 \cdot \mathbb{Q}^8 \cdot \mathbb{Q}^8 \cdot \mathbb{Q}(\sqrt{2})^4 \cdot \mathbb{Q}(\sqrt{2}, \sqrt{15})^8$ | $2, 3, 5$ |
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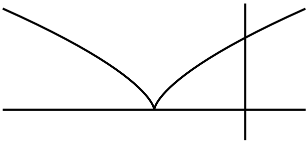
Special fibre at $t = 2$ of minimal regular model of $\mathcal{Q}$ in characteristic $\notin \{3, 5\}$:



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| 2 | $\mathbb{Q}^1 \cdot \mathbb{Q}^2 \cdot \mathbb{Q}^4 \cdot \mathbb{Q}^8 \cdot \mathbb{Q}^8 \cdot \mathbb{Q}(\sqrt{2})^4 \cdot \mathbb{Q}(\sqrt{2}, \sqrt{15})^8$ | $2, 3, 5$ |
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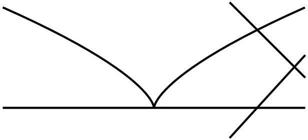
Special fibre at $t = 2$ of minimal regular model of $\mathcal{Q}$ in characteristic 5:



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| $t$ | Place decomposition | Ramification |
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| $1$ | $\mathbb{Q}(\sqrt{3})^1 \cdot \mathbb{Q}(\sqrt{-1})^3 \cdot \left(\mathbb{Q}(\zeta_9)^+(\sqrt{-1})\right)^9 \cdot \left(\mathbb{Q}(\zeta_{36})^+\right)^3$ | $2, 3$ |
| $-1$ | $\mathbb{Q}(\sqrt{-21})^1 \cdot K_6^1 \cdot K_{18}^1 \cdot K_{18}'^{\,3}$ | $2, 3, 7, 11$ |
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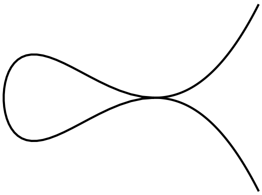
Special fibre at $t = \infty$ of (minimal regular model of) $\mathcal{H}$ in any characteristic:



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$$\mathcal{H} : y^2 = x^6 - x^4 + (t-1)(x^2+x), \quad \ell = 3.$$

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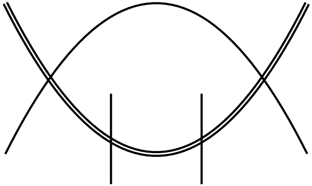
Special fibre at $t = \infty$ of minimal regular model of base change of $\mathcal{H}$ to $\mathbb{Q}(t^{1/2})$ in characteristic $\neq 2$:



This model is no longer regular in characteristic 2.

# Any questions?

Thank you!