CHEAT SHEET FOR L-FUNCTIONS

KARIM BELABAS

1. Constructors

`lfuncreate(1)`	Riemann ζ
`lfuncreate(D:int)`	Dirichlet $L((D/.), s)$
`lfuncreate(Mod(m,q))`	Dirichlet $L(\chi_q(m,\cdot), s)$, $m \in (\mathbb{Z}/q\mathbb{Z})^\times$
`lfuncreate(pol)`	Dedekind ζ_K
`lfuncreate(ell)`	Hasse-Weil ζ (elliptic curve)

Dirichlet L-function.

```
? N = 100; G = idealstar(,N);  \\ (Z/100Z)^*
? G.cyc
%2 = [20, 2]
? char = [1, 0]  \\ g_1 -> e(1/20), g_2 -> e(0/2)
? L = lfuncreate([G, char])
```

Hecke L-function.

```
T = y^2 - 2; K = bnfinit(T, 1);  \\ K = Q(sqrt(2))
mod = [100, [1,1]];
bnr = bnrinit(K, mod, 1);
char = [20, 1, 1];
L = lfuncreate([bnr, char])
```

Other constructors. `lfunartin` (Artin L function), `lfunetaquo` (η quotients), `lfunqf` (theta function of lattices).

Heavy Duty Initialization.

```
lfuninit(L, [c, w, h])  \\ |Re(s - c)| <= w, |Im(s - c)| <= h
lfuninit(L, [c, w, h], m)  \\ up to m-th derivative
lfunthetainit(L, rho)  \\ t is real, t >= rho
```

2. Values

`lfun(L, s)`	$L(s)$
`lfun(L, s, m)`	$L^{(m)}(s)$, m-th derivative
`lfunlambda(L, s, m)`	$\Lambda^{(m)}(s)$, m-th derivative
`lfuntheta(L, t, m)`	$\theta^{(m)}(t)$

3. Miscellaneous

`lfunan(L, n)`	$[a_1, \dots, a_n]$
`lfunzeros(L, H)`	zeros of $L(k/2+it)$, $0 \leqslant t \leqslant H$
`lfunzeros(L, [m,M])`	zeros of $L(k/2+it)$, $m \leqslant t \leqslant M$
`lfunorderzero(L)`	$\mathrm{ord}_{s=k/2} L(s)$
`lfunhardy(L, t)`	$Z(t)$, real function vanishing iff $L(k/2+it) = 0$
`lfunconductor(L)`	guess conductor
`lfuncheckfeq(L)`	consistency check via functional equation