

Modular forms, modular symbols

(PARI-GP version 2.9.0)

Modular Forms

To be completed later.

Modular Symbols

Let $G = \Gamma_0(N)$, $V_k = \mathbf{Q}[X, Y]_{k-2}$. We let $\Delta = \text{Div}^0(\mathbf{P}^1(\mathbf{Q}))$; an element of Δ is a *path* between cusps of $X_0(N)$ via the identification $[b] - [a] \rightarrow$ the path from a to b . A path is coded by the pair $[a, b]$, where a, b are rationals or ∞ , denoting the point at infinity $(1 : 0)$.

Let $\mathbf{M}_k(G) = \text{Hom}_G(\Delta, V_k) \simeq H_c^1(X_0(G), V_k)$; an element of $\mathbf{M}_k(G)$ is a V_k -valued *modular symbol*. There is a natural decomposition $\mathbf{M}_k(G) = \mathbf{M}_k(G)^+ \oplus \mathbf{M}_k(G)^-$ under the action of the $*$ involution, induced by complex conjugation. The `msinit` function computes either \mathbf{M}_k ($\varepsilon = 0$) or its \pm -parts ($\varepsilon = \pm 1$) and fixes a minimal set of $\mathbf{Z}[G]$ -generators (g_i) of Δ .

initialize $M = \mathbf{M}_k(\Gamma_0(N))^\varepsilon$	<code>msinit(N, k, {$\varepsilon = 0$})</code>
the level M	<code>msgetlevel(M)</code>
the weight k	<code>msgetweight(M)</code>
the sign ε	<code>msgetsign(M)</code>

$\mathbf{Z}[G]$ -generators and relations for Δ	<code>mspathgens(M)</code>
Decompose $p = [a, b]$ on the (g_i)	<code>mspathlog(M, p)</code>

Create a symbol

Eisenstein symbol attached to cusp c	<code>msfromcusp(M, c)</code>
Cuspidal symbol attached to E/\mathbf{Q}	<code>msfromell(E)</code>
symbol having given Hecke eigenvalues	<code>msfromhecke(M, v, {H})</code>
is s a symbol ?	<code>msissymbol(M, s)</code>
the list of all $s(g_i)$	<code>mseval(M, s)</code>
evaluate symbol s on path $p = [a, b]$	<code>mseval(M, s, p)</code>

Operators

An operator is given by a matrix of a fixed \mathbf{Q} -basis. H , if given, is a stable \mathbf{Q} -subspace of $\mathbf{M}_k(G)$: operator is restricted to H .

matrix of Hecke operator T_p or U_p	<code>mshecke(M, p, {H})</code>
matrix of Atkin-Lehner w_Q	<code>msatkinlehner(M, Q, {H})</code>
matrix of the $*$ involution	<code>msstar(M, {H})</code>

Subspaces

A subspace is given by a structure allowing quick projection and restriction of linear operators. Its first component is a matrix with integer coefficients whose columns form a \mathbf{Q} -basis. If H is a Hecke-stable subspace of $M_k(G)^+$ or $M_k(G)^-$, it can be split into a direct sum of Hecke-simple subspaces. To a simple subspace corresponds a single normalized newform $\sum_n a_n q^n$.

cuspidal subspace $S_k(G)^\varepsilon$	<code>mscuspidal(M)</code>
Eisenstein subspace $E_k(G)^\varepsilon$	<code>mseisenstein(M)</code>
new part of $S_k(G)^\varepsilon$	<code>msnew(M)</code>
split H into simple subspaces (of $\dim \leq d$)	<code>mssplit(M, H, {d})</code>
(a_1, \dots, a_B) for attached newform	<code>msqexpansion(M, H, {B})</code>

Overconvergent symbols and p -adic L functions

Let M be a full modular symbol space given by `msinit` and p be a prime. To a classical modular symbol ϕ of level N ($v_p(N) \leq 1$), which is an eigenvector for T_p with non-zero eigenvalue a_p , we can attach a p -adic L -function L_p . The function L_p is defined on continuous characters of $\text{Gal}(\mathbf{Q}(\mu_{p^\infty})/\mathbf{Q})$; in GP we allow characters $\langle \chi \rangle^{s_1} \tau^{s_2}$, where (s_1, s_2) are integers, τ is the Teichmüller character and χ is the cyclotomic character.

The symbol ϕ can be lifted to an *overconvergent* symbol Φ , taking values in spaces of p -adic distributions (represented in GP by a list of moments modulo p^n).

`mspadicinit` precomputes data used to lift symbols. If *flag* is given, it speeds up the computation by assuming that $v_p(a_p) = 0$ if *flag* = 0 (fastest), and that $v_p(a_p) \geq \text{flag}$ otherwise (faster as *flag* increases).

`mspadicmoments` computes distributions *mu* attached to Φ allowing to compute L_p to high accuracy.

initialize M_p to lift symbols	<code>mspadicinit(M, p, n, {<i>flag</i>})</code>
lift symbol ϕ	<code>mstooms(Mp, ϕ)</code>
eval overconvergent symbol Φ on path p	<code>msomseval(Mp, Φ, p)</code>
<i>mu</i> for p -adic L -functions	<code>mspadicmoments(Mp, S, {$D = 1$})</code>
$L_p^{(r)}(\chi^s)$, $s = [s_1, s_2]$	<code>mspadicL(mu, {$s = 0$}, {$r = 0$})</code>
$\hat{L}_p(\tau^i)(x)$	<code>mspadicseries(mu, {$i = 0$})</code>

Based on an earlier version by Joseph H. Silverman

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