Pari-GP reference card

(PARI-GP version 2.15.5)

Note: optional arguments are surrounded by braces {}.
To start the calculator, type its name in the terminal: `gp`
To exit `gp`, type `quit`, `\q`, or `<C-D>` at prompt.

Help

describe function	?*function*
extended description	??*keyword*
list of relevant help topics	???*pattern*
name of GP-1.39 function f in GP-2.*	whatnow(f)

Input/Output

previous result, the result before	%, %`, %``, etc.
n-th result since startup	%n
separate multiple statements on line	;
extend statement on additional lines	\
extend statements on several lines	{seq_1; seq_2;}
comment	/* ... */
one-line comment, rest of line ignored	\\ ...

Metacommands & Defaults

set default d to *val*	default({d}, {*val*})
toggle timer on/off	#
print time for last result	##
print defaults	\d
set debug level to n	\g n
set memory debug level to n	\gm n
set n significant digits / bits	\p n, \pb n
set n terms in series	\ps n
quit GP	\q
print the list of PARI types	\t
print the list of user-defined functions	\u
read file into GP	\r *filename*
set debuglevel for domain D to n	setdebug(D, n)

Debugger / break loop

get out of break loop	break or <C-D>
go up/down n frames	dbg_up({n}), dbg_down
set break point	breakpoint()
examine object o	dbg_x(o)
current error data	dbg_err()
number of objects on heap and their size	getheap()
total size of objects on PARI stack	getstack()

PARI Types & Input Formats

t_INT. Integers; hex, binary	±31; ±0x1F, ±0b101
t_REAL. Reals	±3.14, 6.022 E23
t_INTMOD. Integers modulo m	Mod(n, m)
t_FRAC. Rational Numbers	n/m
t_FFELT. Elt in finite field $\mathbf{F}_q$	ffgen(q,'t)
t_COMPLEX. Complex Numbers	$x + y *$ I
t_PADIC. p-adic Numbers	x + O(p^k)
t_QUAD. Quadratic Numbers	$x + y *$ quadgen(D,{'w})
t_POLMOD. Polynomials modulo g	Mod(f, g)
t_POL. Polynomials	$a * x$^$n + \cdots + b$
t_SER. Power Series	f + O(x^k)
t_RFRAC. Rational Functions	f/g
t_QFB. Binary quadratic form	Qfb(a, b, c)
t_VEC/t_COL. Row/Column Vectors	[x, y, z], [x, y, z]~
t_VEC integer range	[1..10]
t_VECSMALL. Vector of small ints	Vecsmall([x, y, z])
t_MAT. Matrices	[a, b; c, d]
t_LIST. Lists	List([x, y, z])
t_STR. Strings	"abc"
t_INFINITY. $\pm\infty$	+oo, -oo

Reserved Variable Names

$\pi \approx 3.14$, $\gamma \approx 0.57$, $C \approx 0.91$, $I = \sqrt{-1}$	Pi, Euler, Catalan, I
Landau's big-oh notation	O

Information about an Object, Precision

PARI type of object x	type(x)
length of x / size of x in memory	#x, sizebyte(x)
real precision / bit precision of x	precision(x), bitprecision(x)
p-adic, series prec. of x	padicprec(x, p), serprec(x, v)
current dynamic precision	getlocalprec, getlocalbitprec

Operators

basic operations	+, - , *, /, ^, sqr
i←i+1, i←i-1, i←i*j, ...	i++, i--, i*=j,...
Euclidean quotient, remainder	x\/y, x\\y, x%y, divrem(x, y)
shift x left or right n bits	x<<n, x>>n or shift(x, ±n)
multiply by 2^n	shiftmul(x, n)
comparison operators	<=, <, >=, >, ==, !=, ===, lex, cmp
boolean operators (or, and, not)	||, &&, !
bit operations	bitand, bitneg, bitor, bitxor, bitnegimply
maximum/minimum of x and y	max(x, y), min(x, y)
sign of x (gives $-1, 0, 1$)	sign(x)
binary exponent of x	exponent(x)
derivative of f, 2nd derivative, etc.	f', f'', ...
differential operator	diffop(f, v, d, {$n = 1$})
quote operator (formal variable)	'x
assignment	x = *value*
simultaneous assignment $x \leftarrow v[1]$, $y \leftarrow v[2]$	[x,y] = v

Select Components

Caveat: components start at index $n = 1$.

n-th component of x	component(x, n)
n-th component of vector/list x	x[n]
components $a, a+1, \ldots, b$ of vector x	x[a..b]
(m, n)-th component of matrix x	x[m, n]
row m or column n of matrix x	x[m,], x[,n]
numerator/denominator of x	numerator(x), denominator(x)

Random Numbers

random integer/prime in $[0, N[$	random(N), randomprime(N)
get/set random seed	getrand, setrand(s)

Conversions

to vector, matrix, vec. of small ints	Col/Vec,Mat,Vecsmall
to list, set, map, string	List, Set, Map, Str
create (x mod y)	Mod(x, y)
make x a polynomial of v	Pol(x, {v})
variants of Pol *et al.*, in reverse order	Polrev, Vecrev, Colrev
make x a power series of v	Ser(x, {v})
convert x to simplest possible type	simplify(x)
object x with real precision n	precision(x, n)
object x with bit precision n	bitprecision(x, n)
set precision to p digits in dynamic scope	localprec(p)
set precision to p bits in dynamic scope	localbitprec(p)

Character strings

convert to TeX representation	strtex(x)
string from bytes / from format+args	strchr, strprintf
split string / join strings	strsplit, strjoin
convert time t ms. to h, m, s, ms format	strtime(t)

Conjugates and Lifts

conjugate of a number x	conj(x)
norm of x, product with conjugate	norm(x)
L^p norm of x (L^∞ if no p)	normlp(x, {p})
square of L^2 norm of x	norml2(x)
lift of x from Mods and p-adics	lift, centerlift(x)
recursive lift	liftall
lift all t_INT and t_PADIC (→t_INT)	liftint
lift all t_POLMOD (→t_POL)	liftpol

Lists, Sets & Maps

Sets (= row vector with strictly increasing entries w.r.t. cmp)

intersection of sets x and y	setintersect(x, y)
set of elements in x not belonging to y	setminus(x, y)
symmetric difference $x \Delta y$	setdelta(x, y)
union of sets x and y	setunion(x, y)
does y belong to the set x	setsearch(x, y, {*flag*})
set of all $f(x, y)$, $x \in X$, $y \in Y$	setbinop(f, X, Y)
is x a set ?	setisset(x)

Lists. create empty list: L = List()

append x to list L	listput(L, x, {i})
remove i-th component from list L	listpop(L, {i})
insert x in list L at position i	listinsert(L, x, i)
sort the list L in place	listsort(L, {*flag*})

Maps. create empty dictionary: M = Map()

attach value v to key k	mapput(M, k, v)
recover value attach to key k or error	mapget(M, k)
is key k in the dict? (set v to $M(k)$)	mapisdefined(M, k, {&v})
remove k from map domain	mapdelete(M, k)

GP Programming

User functions and closures

x, y are formal parameters; y defaults to Pi if parameter omitted; z, t are local variables (lexical scope), z initialized to 1.

```
fun(x, y=Pi) = my(z=1, t); seq
fun = (x, y=Pi) -> my(z=1, t); seq
```

attach help message h to s	addhelp(s, h)
undefine symbol s (also kills help)	kill(s)

Control Statements (X: formal parameter in expression *seq*)

if $a \neq 0$, evaluate seq_1, else seq_2	if(a, {seq_1}, {seq_2})
eval. *seq* for $a \le X \le b$	for($X = a$, b, *seq*)
...for $X \in v$	foreach(v, X, *seq*)
...for primes $a \le X \le b$	forprime($X = a$, b, *seq*)
...for primes $\equiv a \pmod q$	forprimestep($X = a$, b, q, *seq*)
...for composites $a \le X \le b$	forcomposite($X = a$, b, *seq*)
...for $a \le X \le b$ stepping s	forstep($X = a$, b, s, *seq*)
...for X dividing n	fordiv(n, X, *seq*)
...$X = [n, factor(n)]$, $a \le n \le b$	forfactored($X = a$, b, *seq*)
...as above, n squarefree	forsquarefree($X = a$, b, *seq*)
...$X = [d, factor(d)]$, $d \mid n$	fordivfactored(n, X, *seq*)
multivariable for, lex ordering	forvec($X = v$, *seq*)

loop over partitions of n	`forpart(p = n, seq)`
... permutations of S	`forperm(S, p, seq)`
... subsets of $\{1,\dots,n\}$	`forsubset(n, p, seq)`
... k-subsets of $\{1,\dots,n\}$	`forsubset([n, k], p, seq)`
... vectors v, $q(v) \le B$; $q > 0$	`forqfvec(v, q, b, seq)`
... $H < G$ finite abelian group	`forsubgroup(H = G)`
evaluate *seq* until $a \ne 0$	`until(a, seq)`
while $a \ne 0$, evaluate *seq*	`while(a, seq)`
exit n innermost enclosing loops	`break({n})`
start new iteration of n-th enclosing loop	`next({n})`
return x from current subroutine	`return({x})`

Exceptions, warnings

raise an exception / warning	`error()`, `warning()`
type of error message E	`errname(E)`
try seq_1, evaluate seq_2 on error	`iferr(seq1, E, seq2)`

Functions with closure arguments / results

number of arguments of f	`arity(f)`
select from v according to f	`select(f, v)`
apply f to all entries in v	`apply(f, v)`
evaluate $f(a_1,\dots,a_n)$	`call(f, a)`
evaluate $f(\dots f(f(a_1,a_2),a_3)\dots,a_n)$	`fold(f, a)`
calling function as closure	`self()`

Sums & Products

sum $X = a$ to $X = b$, initialized at x	`sum(X = a, b, expr, {x})`
sum entries of vector v	`vecsum(v)`
product of all vector entries	`vecprod(v)`
sum *expr* over divisors of n	`sumdiv(n, X, expr)`
... assuming *expr* multiplicative	`sumdivmult(n, X, expr)`
product $a \le X \le b$, initialized at x	`prod(X = a, b, expr, {x})`
product over primes $a \le X \le b$	`prodeuler(X = a, b, expr)`

Sorting

sort x by k-th component	`vecsort(x, {k}, {fl = 0})`
min. m of x ($m = x[i]$), max.	`vecmin(x, {&i})`, `vecmax`
does y belong to x, sorted wrt. f	`vecsearch(x, y, {f})`
$\prod g^x \to$ factorization ($\Rightarrow$ sorted, unique g)	`matreduce(m)`

Input/Output

print with/without `\n`, TeX format	`print`, `print1`, `printtex`
pretty print matrix	`printp`
print fields with separator	`printsep(sep,...)`,, `printsep1`
formatted printing	`printf()`
write *args* to file	`write`, `write1`, `writetex(file, args)`
write x in binary format	`writebin(file, x)`
read file into GP	`read({file})`
... return as vector of lines	`readvec({file})`
... return as vector of strings	`readstr({file})`
read a string from keyboard	`input()`

Files and file descriptors

File descriptors allow efficient small consecutive reads or writes from or to a given file. The argument n below is always a descriptor, attached to a file in `r`(ead), `w`(rite) or `a`(ppend) mode.

get descriptor n for file *path* in given *mode*	`fileopen(path, mode)`
... from shell *cmd* output (pipe)	`fileextern(cmd)`
close descriptor	`fileclose(n)`
commit pending write operations	`fileflush(n)`
read logical line from file	`fileread(n)`
... raw line from file	`filereadstr(n)`
write `s\n` to file	`filewrite(n, s)`
... write s to file	`filewrite1(n, s)`

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Timers

CPU time in *ms* and reset timer	`gettime()`
CPU time in *ms* since gp startup	`getabstime()`
time in *ms* since UNIX Epoch	`getwalltime()`
timeout command after s seconds	`alarm(s, expr)`

Interface with system

allocates a new stack of s bytes	`allocatemem({s})`
alias *old* to *new*	`alias(new, old)`
install function from library	`install(f, code, {gpf}, {lib})`
execute system command a	`system(a)`
... and feed result to GP	`extern(a)`
... returning GP string	`externstr(a)`
get `$VAR` from environment	`getenv("VAR")`
expand env. variable in string	`strexpand(x)`

Parallel evaluation

These functions evaluate their arguments in parallel (pthreads or MPI); args. must not access global variables (use `export` for this) and must be free of side effects. Enabled if threading engine is not *single* in gp header.

evaluate f on $x[1],\dots,x[n]$	`parapply(f, x)`
evaluate closures $f[1],\dots,f[n]$	`pareval(f)`
as `select`	`parselect(f, A, {flag})`
as `sum`	`parsum(i = a, b, expr)`
as `vector`	`parvector(n, i, {expr})`
eval f for $i = a,\dots,b$	`parfor(i = a, {b}, f, {r}, {f2})`
... for each element x in v	`parforeach(v, x, f, {r}, {f2})`
... for p prime in $[a, b]$	`parforprime(p = a, {b}, f, {r}, {f2})`
... for $p = a$ mod q	`parforprimestep(p = a, {b}, q, f, {r}, {f2})`
... multivariate	`parforvec(X = v, f, {r}, {f2}, {flag})`
export x to parallel world	`export(x)`
... all dynamic variables	`exportall()`
frees exported value x	`unexport(x)`
... all exported values	`unexportall()`

Linear Algebra

dimensions of matrix x	`matsize(x)`
multiply two matrices	`x * y`
... assuming result is diagonal	`matmultodiagonal(x, y)`
concatenation of x and y	`concat(x, {y})`
extract components of x	`vecextract(x, y, {z})`
transpose of vector or matrix x	`x~`, `mattranspose(x)`
adjoint of the matrix x	`matadjoint(x)`
eigenvectors/values of matrix x	`mateigen(x)`
characteristic/minimal polynomial of x	`charpoly(x)`, `minpoly(x)`
trace/determinant of matrix x	`trace(x)`, `matdet(x)`
permanent of matrix x	`matpermanent(x)`
Frobenius form of x	`matfrobenius(x)`
QR decomposition	`matqr(x)`
apply `matqr`'s transform to v	`mathouseholder(Q, v)`

Constructors & Special Matrices

$\{g(x) : x \in v$ s.t. $f(x)\}$	`[g(x) | x <- v, f(x)]`
$\{x : x \in v$ s.t. $f(x)\}$	`[x | x <- v, f(x)]`
$\{g(x) : x \in v\}$	`[g(x) | x <- v]`
row vec. of *expr* eval'ed at $1 \le i \le n$	`vector(n, {i}, {expr})`
col. vec. of *expr* eval'ed at $1 \le i \le n$	`vectorv(n, {i}, {expr})`
vector of small ints	`vectorsmall(n, {i}, {expr})`
$[c, c\cdot x, \dots, c\cdot x^n]$	`powers(x, n, {c = 1})`
$[1, 2^x, \dots, n^x]$	`dirpowers(n, x)`
matrix $1 \le i \le m$, $1 \le j \le n$	`matrix(m, n, {i}, {j}, {expr})`
define matrix by blocks	`matconcat(B)`
diagonal matrix with diagonal x	`matdiagonal(x)`
is x diagonal?	`matisdiagonal(x)`
$x \cdot$ `matdiagonal(d)`	`matmuldiagonal(x, d)`
$n \times n$ identity matrix	`matid(n)`
Hessenberg form of square matrix x	`mathess(x)`
$n \times n$ Hilbert matrix $H_{ij} = (i + j - 1)^{-1}$	`mathilbert(n)`
$n \times n$ Pascal triangle	`matpascal(n - 1)`
companion matrix to polynomial x	`matcompanion(x)`
Sylvester matrix of x and y	`polsylvestermatrix(x, y)`

Gaussian elimination

kernel of matrix x	`matker(x, {flag})`
intersection of column spaces of x and y	`matintersect(x, y)`
solve $MX = B$ (M invertible)	`matsolve(M, B)`
one sol of $M * X = B$	`matinverseimage(M, B)`
basis for image of matrix x	`matimage(x)`
columns of x *not* in `matimage`	`matimagecompl(x)`
supplement columns of x to get basis	`matsupplement(x)`
rows, cols to extract invertible matrix	`matindexrank(x)`
rank of the matrix x	`matrank(x)`
solve $MX = B$ mod D	`matsolvemod(M, D, B)`
image mod D	`matimagemod(M, D)`
kernel mod D	`matkermod(M, D)`
inverse mod D	`matinvmod(M, D)`
determinant mod D	`matdetmod(M, D)`

Lattices & Quadratic Forms

Quadratic forms

evaluate ${}^t x Q y$	`qfeval({Q = id}, x, y)`
evaluate ${}^t x Q x$	`qfeval({Q = id}, x)`
signature of quad form ${}^t y * x * y$	`qfsign(x)`
decomp into squares of ${}^t y * x * y$	`qfgaussred(x)`
eigenvalues/vectors for real symmetric x	`qfjacobi(x)`

HNF and SNF

upper triangular Hermite Normal Form	`mathnf(x)`
HNF of x where d is a multiple of $\det(x)$	`mathnfmod(x, d)`
multiple of $\det(x)$	`matdetint(x)`
HNF of ($x \mid$ `diagonal(D)`)	`mathnfmodid(x, D)`
elementary divisors of x	`matsnf(x)`
q-rank from elementary divisors	`snfrank(v, q)`
elementary divisors of $\mathbf{Z}[a]/(f'(a))$	`poldiscreduced(f)`
integer kernel of x	`matkerint(x)`
$\mathbf{Z}$-module $\leftrightarrow$ $\mathbf{Q}$-vector space	`matrixqz(x, p)`

Lattices

LLL-algorithm applied to columns of x	`qflll(x, {flag})`
... for Gram matrix of lattice	`qflllgram(x, {flag})`
find up to m sols of `qfnorm(x, y)` $\le b$	`qfminim(x, b, m)`
v, $v[i]$:=number of y s.t. `qfnorm(x, y)` $= i$	`qfrep(x, B, {flag})`
perfection rank of x	`qfperfection(x)`
find isomorphism between q and Q	`qfisom(q, Q)`
precompute for isomorphism test with q	`qfisominit(q)`
automorphism group of q	`qfauto(q)`

Send comments and corrections to ⟨Karim.Belabas@math.u-bordeaux.fr⟩

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convert qfauto for GAP/Magma	qfautoexport$(G,\{flag\})$
orbits of V under $G \subset \mathrm{GL}(V)$	qforbits(G,V)

Polynomials & Rational Functions

all defined polynomial variables	variables()
get var. of highest priority (higher than v)	varhigher$(name,\{v\})$
... of lowest priority (lower than v)	varlower$(name,\{v\})$

Coefficients, variables and basic operators

degree of f	poldegree(f)
coef. of degree n of f, leading coef.	polcoef(f,n), pollead
main variable / all variables in f	variable(f), variables(f)
replace x by y in f	subst(f,x,y)
evaluate f replacing vars by their value	eval(f)
replace polynomial expr. $T(x)$ by y in f	substpol(f,T,y)
replace $x_1,\dots,x_n$ by $y_1,\dots,y_n$ in f	substvec(f,x,y)
$f \in A[x]$; reciprocal polynomial $x^{\deg f} f\left(\frac{1}{x}\right)$	polrecip(f)
gcd of coefficients of f	content(f)
derivative of f w.r.t. x	deriv$(f,\{x\})$
... n-th derivative of f	derivn$(f,n,\{x\})$
formal integral of f w.r.t. x	intformal$(f,\{x\})$
formal sum of f w.r.t. x	sumformal$(f,\{x\})$

Constructors & Special Polynomials

interpolation polynomial at $(x[1],y[1]),\dots,(x[n],y[n])$, evaluated at t, with error estimate e	polinterpolate$(x,\{y\},\{t\},\{\&e\})$
T_n/U_n, H_n	polchebyshev(n), polhermite(n)
P_n, $L_n^{(\alpha)}$	pollegendre(n), pollaguerre(n,a)
n-th cyclotomic polynomial Φ_n	polcyclo(n)
return n if $f=\Phi_n$, else 0	poliscyclo(f)
is f a product of cyclotomic polynomials?	poliscycloprod(f)
Zagier's polynomial of index (n,m)	polzagier(n,m)

Resultant, elimination

discriminant of polynomial f	poldisc(f)
find factors of poldisc(f)	poldiscfactors(f)
resultant $R=\mathrm{Res}_v(f,g)$	polresultant$(f,g,\{v\})$
$[u,v,R]$, $xu+yv=\mathrm{Res}_v(f,g)$	polresultantext$(x,y,\{v\})$
solve Thue equation $f(x,y)=a$	thue$(t,a,\{sol\})$
initialize t for Thue equation solver	thueinit(f)

Roots and Factorization (Complex/Real)

complex roots of f	polroots(f)
bound complex roots of f	polrootsbound(f)
number of real roots of f (in $[a,b]$)	polsturm$(f,\{[a,b]\})$
real roots of f (in $[a,b]$)	polrootsreal$(f,\{[a,b]\})$
complex embeddings of t_POLMOD z	conjvec(z)

Roots and Factorization (Finite fields)

factor f mod p, roots	factormod(f,p), polrootsmod
factor f over $\mathbf{F}_p[x]/(T)$, roots	factormod$(f,[T,p])$, polrootsmod
squarefree factorization of f in $\mathbf{F}_q[x]$	factormodSQF$(f,\{D\})$
distinct degree factorization of f in $\mathbf{F}_q[x]$	factormodDDF$(f,\{D\})$
factor n-th cyclotomic pol. Φ_n mod p	factormodcyclo(n,p)

Roots and Factorization (p-adic fields)

factor f over $\mathbf{Q}_p$, roots	factorpadic(f,p,r), polrootspadic
p-adic root of f congruent to a mod p	padicappr(f,a)
Newton polygon of f for prime p	newtonpoly(f,p)
Hensel lift $A/\mathrm{lc}(A)=\prod_i B[i]$ mod p^e	polhensellift(A,B,p,e)
$T=\prod(x-z_i)\mapsto\prod(x-\omega(z_i))]\in\mathbf{Z}_p[x]$	polteichmuller(T,p,e)
extensions of $\mathbf{Q}_p$ of degree N	padicfields(p,N)

Roots and Factorization (Miscellaneous)

symmetric powers of roots of f up to n	polsym(f,n)
Graeffe transform of f, $g(x^2)=f(x)f(-x)$	polgraeffe(f)
factor f over coefficient field	factor(f)
cyclotomic factors of $f\in\mathbf{Q}[X]$	polcyclofactors(f)

Finite Fields

A finite field is encoded by any element (t_FFELT).

find irreducible $T\in\mathbf{F}_p[x]$, $\deg T=n$	ffinit$(p,n,\{x\})$
Create t in $\mathbf{F}_q\simeq\mathbf{F}_p[t]/(T)$	t = ffgen$(T,$'t$)$
... indirectly, with implicit T	t = ffgen$(q,$'t$)$; T = t.mod
map m from $\mathbf{F}_q\ni a$ to $\mathbf{F}_{q^k}\ni b$	m = ffembed(a,b)
build $K=\mathbf{F}_q[x]/(P)$ extending $\mathbf{F}_q\ni a$,	ffextend(a,P)
evaluate map m on x	ffmap(m,x)
inverse map of m	ffinvmap(m)
compose maps $m\circ n$	ffcompomap(m,n)
x as polmod over codomain of map m	ffmaprel(m,x)
F^n over $\mathbf{F}_q\ni a$	fffrobenius(a,n)
$\#\{$monic irred. $T\in\mathbf{F}_q[x],\deg T=n\}$	ffnbirred(q,n)

Formal & p-adic Series

truncate power series or p-adic number	truncate(x)
valuation of x at p	valuation(x,p)

Dirichlet and Power Series

Taylor expansion around 0 of f w.r.t. x	taylor(f,x)
Laurent series of closure F up to x^k	laurentseries(f,k)
$\sum a_k b_k t^k$ from $\sum a_k t^k$ and $\sum b_k t^k$	serconvol(a,b)
$f=\sum a_k t^k$ from $\sum (a_k/k!)t^k$	serlaplace(f)
reverse power series F so $F(f(x))=x$	serreverse(f)
remove terms of degree $<n$ in f	serchop(f,n)
Dirichlet series multiplication / division	dirmul, dirdiv(x,y)
Dirichlet Euler product (b terms)	direuler$(p=a,b,expr)$

Transcendental and p-adic Functions

real, imaginary part of x	real(x), imag(x)
absolute value, argument of x	abs(x), arg(x)
square/nth root of x	sqrt(x), sqrtn$(x,n,\{\&z\})$
all n-th roots of 1	rootsof1(n)
FFT of $[f_0,\dots,f_{n-1}]$	w = fftinit(n), fft/fftinv(w,f)
trig functions	sin, cos, tan, cotan, sinc
inverse trig functions	asin, acos, atan
hyperbolic functions	sinh, cosh, tanh, cotanh
inverse hyperbolic functions	asinh, acosh, atanh
$\log(x)$, $\log(1+x)$, e^x, e^x-1	log, log1p, exp, expm1
Euler Γ function, $\log\Gamma$, Γ'/Γ	gamma, lngamma, psi
half-integer gamma function $\Gamma(n+1/2)$	gammah(n)
Riemann's zeta $\zeta(s)=\sum n^{-s}$	zeta(s)
$\sum_{1\le n\le N} n^s$	dirpowerssum(N,s)
Hurwitz's $\zeta(s,x)=\sum(n+x)^{-s}$	zetahurwitz(s,x)
Lerch $\Phi(z,s,x)=\sum z^n(n+x)^{-s}$	lerchphi(z,s,x)
Lerch $L(s,x,t)=\Phi(e^{2i\pi t},s,x)$	lerchzeta(s,x,t)
multiple zeta value (MZV), $\zeta(s_1,\dots,s_k)$	zetamult$(s,\{T\})$
all MZVs for weight $\sum s_i=n$	zetamultall(n)
convert MZV id to $[s_1,\dots,s_k]$	zetamultconvert$(f,\{flag\})$
MZV dual sequence	zetamultdual(s)
multiple polylog $Li_{s_1,\dots s_k}(z_1,\dots,z_k)$	polylogmult(s,z)
incomplete Γ function ($y=\Gamma(s)$)	incgam$(s,x,\{y\})$
complementary incomplete Γ	incgamc(s,x)
$\int_x^\infty e^{-t}\,dt/t$, $(2/\sqrt{\pi})\int_x^\infty e^{-t^2}dt$	eint1, erfc
elliptic integral of 1st and 2nd kind	ellK(k), ellE(k)
dilogarithm of x	dilog(x)
m-th polylogarithm of x	polylog$(m,x,\{flag\})$
U-confluent hypergeometric function	hyperu(a,b,u)
Hypergeometric ${}_pF_q(A,B;z)$	hypergeom(A,B,z)
Bessel $J_n(x)$, $J_{n+1/2}(x)$	besselj(n,x), besseljh(n,x)
Bessel I_ν, K_ν, H^1_ν, H^2_ν, Y_ν	(bessel)i, k, h1, h2, y
k-th zero of $J_\nu(x)$	besseljzero$(nu,\{k=1\})$
k-th zero of $Y_\nu(x)$	besselyzero$(nu,\{k=1\})$
Airy functions $A_i(x)$, $B_i(x)$	airy(x)
Lambert W: x s.t. $xe^x=y$	lambertw(y)
Teichmuller character of p-adic x	teichmuller(x)

Iterations, Sums & Products

Numerical integration for meromorphic functions

Behaviour at endpoint for Double Exponential (DE) methods: either a scalar ($a\in\mathbf{C}$, regular) or $\pm$oo (decreasing at least as x^{-2}) or

$(x-a)^{-\alpha}$ singularity	$[a,\alpha]$
exponential decrease $e^{-\alpha\lvert x\rvert}$	$[\pm\infty,\alpha]$, $\alpha>0$
slow decrease $\lvert x\rvert^\alpha$	... $\alpha<-1$
oscillating as $\cos(kx)$)	$\alpha=k$I, $k>0$
oscillating as $\sin(kx)$)	$\alpha=-k$I, $k>0$
numerical integration	intnum$(x=a,b,f,\{T\})$
weights T for intnum	intnuminit$(a,b,\{m\})$
weights T incl. kernel K	intfuncinit$(t=a,b,K,\{m\})$
integrate $(2i\pi)^{-1}f$ on circle $\lvert z-a\rvert=R$	intcirc$(x=a,R,f,\{T\})$

Other integration methods

n-point Gauss-Legendre	intnumgauss$(x=a,b,f,\{n\})$
weights for n-point Gauss-Legendre	intnumgaussinit$(\{n\})$
quasi-periodic function, period $2H$	intnumosc$(x=a,f,H)$
Romberg (low accuracy)	intnumromb$(x=a,b,f,\{flag\})$

Numerical summation

sum of series $f(n)$, $n\ge a$ (low accuracy)	suminf$(n=a,expr)$
sum of alternating/positive series	sumalt, sumpos
sum of series using Euler-Maclaurin	sumnum$(n=a,f,\{T\})$
... Sidi summation	sumnumsidi$(n=a,f)$
$\sum_{n\ge a}F(n)$, F rational function	sumnumrat(F,a)
... $\sum_{p\ge a}F(p^s)$	sumeulerrat$(F,\{s=1\},\{a=2\})$
weights for sumnum, a as in DE	sumnuminit$(\{\infty,a\})$
sum of series by Monien summation	sumnummonien$(n=a,f,\{T\})$
weights for sumnummonien	sumnummonieninit$(\{\infty,a\})$
sum of series using Abel-Plana	sumnumap$(n=a,f,\{T\})$
weights for sumnumap, a as in DE	sumnumapinit$(\{\infty,a\})$
sum of series using Lagrange	sumnumlagrange$(n=a,f,\{T\})$
weights for sumnumlagrange	sumnumlagrangeinit

Products

product $a\le X\le b$, initialized at x	prod$(X=a,b,expr,\{x\})$
product over primes $a\le X\le b$	prodeuler$(X=a,b,expr)$
infinite product $a\le X\le\infty$	prodinf$(X=a,expr)$
$\prod_{n\ge a}F(n)$, F rational function	prodnumrat(F,a)
$\prod_{p\ge a}F(p^s)$	prodeulerrat$(F,\{s=1\},\{a=2\})$

Pari-GP reference card

(PARI-GP version 2.15.5)

Other numerical methods

real root of f in $[a, b]$; bracketed root	solve$(X = a, b, f)$
... interval splitting, step s	solvestep$(X = a, b, s, f, \{flag = 0\})$
limit of $f(t)$, $t \to \infty$	limitnum(f, {alpha})
asymptotic expansion of f (rational)	asympnum(f, {alpha})
... $N + 1$ terms as floats	asympnumraw(f, N, {alpha})
numerical derivation w.r.t x: $f'(a)$	derivnum$(x = a, f)$
evaluate continued fraction F at t	contfraceval$(F, t, \{L\})$
power series to cont. fraction (L terms)	contfracinit$(S, \{L\})$
Padé approximant (deg. denom. $\leq B$)	bestapprPade$(S, \{B\})$

Elementary Arithmetic Functions

vector of binary digits of $\lvert x \rvert$	binary(x)
bit number n of integer x	bittest(x, n)
Hamming weight of integer x	hammingweight(x)
digits of integer x in base B	digits$(x, \{B = 10\})$
sum of digits of integer x in base B	sumdigits$(x, \{B = 10\})$
integer from digits	fromdigits$(v, \{B = 10\})$
ceiling/floor/fractional part	ceil, floor, frac
round x to nearest integer	round$(x, \{\&e\})$
truncate x	truncate$(x, \{\&e\})$
gcd/LCM of x and y	gcd(x, y), lcm(x, y)
gcd of entries of a vector/matrix	content(x)

Primes and Factorization

extra prime table	addprimes()
add primes in v to prime table	addprimes(v)
remove primes from prime table	removeprimes(v)
Chebyshev $\pi(x)$, n-th prime p_n	primepi(x), prime(n)
vector of first n primes	primes(n)
smallest prime $\geq x$	nextprime(x)
largest prime $\leq x$	precprime(x)
factorization of x	factor$(x, \{lim\})$
... selecting specific algorithms	factorint$(x, \{flag = 0\})$
$n = df^2$, d squarefree/fundamental	core$(n, \{fl\})$, coredisc
certificate for (prime) N	primecert(N)
verifies a certificate c	primecertisvalid(c)
convert certificate to Magma/PRIMO	primecertexport
recover x from its factorization	factorback$(f, \{e\})$
$x \in \mathbf{Z}$, $\lvert x \rvert \leq X$, $\gcd(N, P(x)) \geq N$	zncoppersmith$(P, N, X, \{B\})$
divisors of N in residue class r mod s	divisorslenstra(N, r, s)

Divisors and multiplicative functions

number of prime divisors $\omega(n)$ / $\Omega(n)$	omega(n), bigomega
divisors of n / number of divisors $\tau(n)$	divisors(n), numdiv
sum of (k-th powers of) divisors of n	sigma$(n, \{k\})$
Möbius μ-function	moebius(x)
Ramanujan's τ-function	ramanujantau(x)

Combinatorics

factorial of x	x! or factorial(x)
binomial coefficient $\binom{x}{k}$	binomial$(x, \{k\})$
Bernoulli number B_n as real/rational	bernreal(n), bernfrac
$[B_0, B_2, \ldots B_{2k}]$	bernvec(k)
Bernoulli polynomial $B_n(x)$	bernpol$(n, \{x\})$
Euler numbers	eulerfrac, eulerreal, eulervec
Euler polynomial $E_n(x)$	eulerpol$(n, \{x\})$
Eulerian polynomial $A_n(x)$	eulerianpol
Fibonacci number F_n	fibonacci(n)
Harmonic number $H_{n,r} = 1^{-r} + \ldots + n^{-r}$	harmonic(n, r)
Stirling numbers $s(n, k)$ and $S(n, k)$	stirling$(n, k, \{flag\})$

number of partitions of n	numbpart(n)
k-th permutation on n letters	numtoperm(n, k)
... index k of permutation v	permtonum(v)
order of permutation p	permorder(p)
signature of permutation p	permsign(p)
cyclic decomposition of permutation p	permcycles(p)

Multiplicative groups $(\mathbf{Z}/N\mathbf{Z})^*$, $\mathbf{F}_q^*$

Euler ϕ-function	eulerphi(x)
multiplicative order of x (divides o)	znorder$(x, \{o\})$, fforder
primitive root mod q / x.mod	znprimroot(q), ffprimroot(x)
structure of $(\mathbf{Z}/n\mathbf{Z})^*$	znstar(n)
discrete logarithm of x in base g	znlog$(x, g, \{o\})$, fflog
Kronecker-Legendre symbol $(\frac{x}{y})$	kronecker(x, y)
quadratic Hilbert symbol (at p)	hilbert$(x, y, \{p\})$

Euclidean algorithm, continued fractions

CRT: solve $z \equiv x$ and $z \equiv y$	chinese(x, y)
minimal u, v so $xu + yv = \gcd(x, y)$	gcdext(x, y)
half-gcd algorithm	halfgcd(x, y)
continued fraction of x	contfrac$(x, \{b\}, \{lmax\})$
last convergent of continued fraction x	contfracpnqn(x)
rational approximation to x (den. $\leq B$)	bestappr$(x, \{B\})$
recognize $x \in \mathbf{C}$ as polmod mod $T \in \mathbf{Z}[X]$	bestapprnf(x, T)

Miscellaneous

integer square / n-th root of x	sqrtint(x), sqrtnint(x, n)
largest integer e s.t. $b^e \leq b$, $e = \lfloor \log_b(x) \rfloor$	logint$(x, b, \{\&z\})$

Characters

Let $cyc = [d_1, \ldots, d_k]$ represent an abelian group $G = \oplus(\mathbf{Z}/d_j\mathbf{Z}) \cdot g_j$ or any structure G affording a .cyc method; e.g. znstar$(q, 1)$ for Dirichlet characters. A character χ is coded by $[c_1, \ldots, c_k]$ such that $\chi(g_j) = e(n_j/d_j)$.

$\chi \cdot \psi$; χ^{-1}; $\chi \cdot \psi^{-1}$; χ^k	charmul, charconj, chardiv, charpow
order of χ	charorder(cyc, χ)
kernel of χ	charker(cyc, χ)
$\chi(x)$, G a GP group structure	chareval$(G, \chi, x, \{z\})$
Galois orbits of characters	chargalois(G)

Dirichlet Characters

initialize $G = (\mathbf{Z}/q\mathbf{Z})^*$	G = znstar$(q, 1)$
convert datum D to $[G, \chi]$	znchar(D)
is χ odd?	zncharisodd(G, χ)
real $\chi \to$ Kronecker symbol $(D/.)$	znchartokronecker(G, χ)
conductor of χ	znchaconductor(G, χ)
$[G_0, \chi_0]$ primitive attached to χ	znchartoprimitive(G, χ)
induce $\chi \in \hat{G}$ to $\mathbf{Z}/N\mathbf{Z}$	zncharinduce(G, χ, N)
χ_p	znchardecompose(G, χ, p)
$\prod_{p \mid (Q,N)} \chi_p$	znchardecompose(G, χ, Q)
complex Gauss sum $G_a(\chi)$	znchargauss(G, χ)

Conrey labelling

Conrey label $m \in (\mathbf{Z}/q\mathbf{Z})^* \to$ character	znconreychar(G, m)
character $\to$ Conrey label	znconreyexp(G, χ)
log on Conrey generators	znconreylog(G, m)
conductor of χ (χ_0 primitive)	znconreyconductor$(G, \chi, \{\chi_0\})$

True-False Tests

is x the disc. of a quadratic field?	isfundamental(x)
is x a prime?	isprime(x)
is x a strong pseudo-prime?	ispseudoprime(x)
is x square-free?	issquarefree(x)
is x a square?	issquare$(x, \{\&n\})$
is x a perfect power?	ispower$(x, \{k\}, \{\&n\})$
is x a perfect power of a prime? ($x = p^n$)	isprimepower$(x, \&n)$
... of a pseudoprime?	ispseudoprimepower$(x, \&n)$
is x powerful?	ispowerful(x)
is x a totient? ($x = \varphi(n)$)	istotient$(x, \{\&n\})$
is x a polygonal number? ($x = P(s, n)$)	ispolygonal$(x, s, \{\&n\})$
is pol irreducible?	polisirreducible(pol)

Graphic Functions

crude graph of $expr$ between a and b	plot$(X = a, b, expr)$

High-resolution plot (immediate plot)

plot $expr$ between a and b	ploth$(X = a, b, expr, \{flag\}, \{n\})$
plot points given by lists lx, ly	plothraw$(lx, ly, \{flag\})$
terminal dimensions	plothsizes()

Rectwindow functions

init window w, with size x,y	plotinit(w, x, y)
erase window w	plotkill(w)
copy w to w_2 with offset (dx, dy)	plotcopy(w, w_2, dx, dy)
clips contents of w	plotclip(w)
scale coordinates in w	plotscale(w, x_1, x_2, y_1, y_2)
ploth in w	plotrecth$(w, X = a, b, expr, \{flag\}, \{n\})$
plothraw in w	plotrecthraw$(w, data, \{flag\})$
draw window w_1 at (x_1, y_1), ...	plotdraw$([[w_1, x_1, y_1], \ldots])$

Low-level Rectwindow Functions

set current drawing color in w to c	plotcolor(w, c)
current position of cursor in w	plotcursor(w)
write s at cursor's position	plotstring(w, s)
move cursor to (x, y)	plotmove(w, x, y)
move cursor to $(x + dx, y + dy)$	plotrmove(w, dx, dy)
draw a box to (x_2, y_2)	plotbox(w, x_2, y_2)
draw a box to $(x + dx, y + dy)$	plotrbox(w, dx, dy)
draw polygon	plotlines$(w, lx, ly, \{flag\})$
draw points	plotpoints(w, lx, ly)
draw line to $(x + dx, y + dy)$	plotrline(w, dx, dy)
draw point $(x + dx, y + dy)$	plotrpoint(w, dx, dy)

Convert to Postscript or Scalable Vector Graphics

The format f is either "ps" or "svg".

as ploth	plothexport$(f, X = a, b, expr, \{flag\}, \{n\})$
as plothraw	plothrawexport$(f, lx, ly, \{flag\})$
as plotdraw	plotexport$(f, [[w_1, x_1, y_1], \ldots])$

Send comments and corrections to ⟨Karim.Belabas@math.u-bordeaux.fr⟩