

Algebraic Number Theory

(PARI-GP version 2.12.1)

Binary Quadratic Forms

create $ax^2 + bxy + cy^2$ (distance d) `qfb(a, b, c, {d})`
 reduce x ($s = \sqrt{D}$, $l = \lfloor s \rfloor$) `qfbred(x, {flag}, {D}, {l}, {s})`
 return $[y, g]$, $g \in \text{SL}_2(\mathbf{Z})$, $y = g \cdot x$ reduced `qfbreds12(x)`
 composition of forms $x*y$ or `qfbnucomp(x, y, l)`
 n -th power of form x^n or `qfbnupow(x, n)`
 composition without reduction `qfbcomprow(x, y)`
 n -th power without reduction `qfbpowrow(x, n)`
 prime form of disc. x above prime p `qfbprimeform(x, p)`
 class number of disc. x `qfbclassno(x)`
 Hurwitz class number of disc. x `qfbhclassno(x)`
 solve $Q(x, y) = n$ in integers `qfbsolve(Q, n)`

Quadratic Fields

quadratic number $\omega = \sqrt{x}$ or $(1 + \sqrt{x})/2$ `quadgen(x)`
 minimal polynomial of ω `quadpoly(x)`
 discriminant of $\mathbf{Q}(\sqrt{x})$ `quaddisc(x)`
 regulator of real quadratic field `quadregulator(x)`
 fundamental unit in real $\mathbf{Q}(\sqrt{D})$ `quadunit(D, {'w'})`
 class group of $\mathbf{Q}(\sqrt{D})$ `quadclassunit(D, {flag}, {t})`
 Hilbert class field of $\mathbf{Q}(\sqrt{D})$ `quadhilbert(D, {flag})`
 ... using specific class invariant ($D < 0$) `polclass(D, {inv})`
 ray class field modulo f of $\mathbf{Q}(\sqrt{D})$ `quadray(D, f, {flag})`

General Number Fields: Initializations

The number field $K = \mathbf{Q}[X]/(f)$ is given by irreducible $f \in \mathbf{Q}[X]$. We denote $\theta = \bar{X}$ the canonical root of f in K . A nf structure contains a maximal order and allows operations on elements and ideals. A bnf adds class group and units. A bnr is attached to ray class groups and class field theory. A rmf is attached to relative extensions L/K .

init number field structure nf `nfinit(f, {flag})`
 known integer basis B `nfinit([f, B])`
 order maximal at $vp = [p_1, \dots, p_k]$ `nfinit([f, vp])`
 order maximal at all $p \leq P$ `nfinit([f, P])`
 certify maximal order `nfcertify(nf)`

nf members:

a monic $F \in \mathbf{Z}[X]$ defining K $nf.pol$
 number of real/complex places $nf.r1/r2/sign$
 discriminant of nf $nf.disc$
 primes ramified in nf $nf.p$
 T_2 matrix $nf.t2$
 complex roots of F $nf.roots$
 integral basis of \mathbf{Z}_K as powers of θ $nf.zk$
 different/codifferent $nf.diff, nf.codiff$
 index $[\mathbf{Z}_K : \mathbf{Z}[X]/(F)]$ $nf.index$
 recompute nf using current precision `nfnewprec(nf)`
 init relative rmf $L = K[Y]/(g)$ `rmfinit(nf, g)`
 init bnf structure `bnfinit(f, 1)`

bnf members: same as nf , plus

underlying nf $bnf.nf$
 class group, regulator $bnf.clgp, bnf.reg$
 fundamental/torsion units $bnf.fu, bnf.tu$
 add S -class group and units, yield $bnfS$ `bnfsunit(bnf, S)`

init class field structure bnr `bnrinit(bnf, m, {flag})`
bnr members: same as bnf , plus
 underlying bnf $bnr.bnf$
 big ideal structure $bnr.bid$
 modulus m $bnr.mod$
 structure of $(\mathbf{Z}_K/m)^*$ $bnr.zkst$

Fields, subfields, embeddings

Defining polynomials, embeddings
 smallest poly defining $f = 0$ (slow) `polredabs(f, {flag})`
 small poly defining $f = 0$ (fast) `polredbest(f, {flag})`
 random Tschirnhausen transform of f `poltschirnhaus(f)`
 $\mathbf{Q}[t]/(f) \subset \mathbf{Q}[t]/(g)$? Isomorphic? `nfisincl(f, g), nfisisom`
 reverse polmod $a = A(t) \bmod T(t)$ `modreverse(a)`
 compositum of $\mathbf{Q}[t]/(f), \mathbf{Q}[t]/(g)$ `polcompositum(f, g, {flag})`
 compositum of $K[t]/(f), K[t]/(g)$ `nfcompositum(nf, f, g, {flag})`
 splitting field of K (degree divides d) `nfsplitting(nf, {d})`
 signs of real embeddings of x `nfeltsign(nf, x, {pl})`
 complex embeddings of x `nfeltembed(nf, x, {pl})`
 $T \in K[t]$, # of real roots of $\sigma(T) \in R[t]$ `nfpolsturm(nf, T, {pl})`

Subfields, polynomial factorization

subfields (of degree d of nf) `nfsubfields(nf, {d})`
 maximal subfields of nf `nfsubfieldsmax(nf)`
 maximal CM subfield of nf `nfsubfieldscm(nf)`
 d -th degree subfield of $\mathbf{Q}(\zeta_n)$ `polsubcyclo(n, d, {v})`
 roots of unity in nf `nfroots of 1(nf)`
 roots of g belonging to nf `nfroots(nf, g)`
 factor g in nf `nfactor(nf, g)`

Linear and algebraic relations

poly of degree $\leq k$ with root $x \in \mathbf{C}$ `algdep(x, k)`
 alg. dep. with pol. coeffs for series s `seralgdep(s, x, y)`
 small linear rel. on coords of vector x `lindep(x)`

Basic Number Field Arithmetic (nf)

Number field elements are `t_INT`, `t_FRAC`, `t_POL`, `t_POLMOD`, or `t_COL` (on integral basis $nf.zk$).

Basic operations

$x + y$ `nfeltadd(nf, x, y)`
 $x \times y$ `nfeltmul(nf, x, y)`
 $x^n, n \in \mathbf{Z}$ `nfeltpow(nf, x, n)`
 x/y `nfeltdiv(nf, x, y)`
 $q = x \setminus y := \text{round}(x/y)$ `nfeltdivceuc(nf, x, y)`
 $r = x \% y := x - (x \setminus y)y$ `nfeltmod(nf, x, y)`
 ... $[q, r]$ as above `nfeltdivrem(nf, x, y)`
 reduce x modulo ideal A `nfeltreduce(nf, x, A)`
 absolute trace $\text{Tr}_{K/\mathbf{Q}}(x)$ `nfelttrace(nf, x)`
 absolute norm $N_{K/\mathbf{Q}}(x)$ `nfeltnorm(nf, x)`

Multiplicative structure of K^* ; $K^*/(K^*)^n$

valuation $v_p(x)$ `nfeltval(nf, x, p)`
 ... write $x = \pi^{v_p(x)}y$ `nfeltval(nf, x, p, &y)`
 quadratic Hilbert symbol (at p) `nfhilbert(nf, a, b, {p})`
 b such that $xb^n = v$ is small `idealredmodpower(nf, x, n)`

Maximal order and discriminant

integral basis of field $\mathbf{Q}[x]/(f)$ `nfbasis(f)`
 field discriminant of $\mathbf{Q}[x]/(f)$ `nfdisc(f)`
 ... and factorization `nfdiscfactors(f)`
 express x on integer basis `nfalgtobasis(nf, x)`
 express element x as a polmod `nfbasistoalg(nf, x)`

Dedekind Zeta Function ζ_K , Hecke L series

$R = [c, w, h]$ in initialization means we restrict $s \in \mathbf{C}$ to domain $|\Re(s) - c| < w$, $|\Im(s)| < h$; $R = [w, h]$ encodes $[1/2, w, h]$ and $[h]$ encodes $R = [1/2, 0, h]$ (critical line up to height h).

ζ_K as Dirichlet series, $N(I) < b$ `dirzetak(nf, b)`
 init $\zeta_K^{(k)}(s)$ for $k \leq n$ `L = lfunitinit(bnf, R, {n = 0})`
 compute $\zeta_K(s)$ (n -th derivative) `lfun(L, s, {n = 0})`
 compute $\Lambda_K(s)$ (n -th derivative) `lfunlambda(L, s, {n = 0})`

init $\zeta_K^{(k)}(s, \chi)$ for $k \leq n$ `L = lfunitinit([bnr, chi], R, {n = 0})`
 compute $L_K(s, \chi)$ (n -th derivative) `lfun(L, s, {n})`
 Artin root number of K `bnrrootnumber(bnr, chi, {flag})`
 $L(1, \chi)$, for all χ trivial on H `bnrL1(bnr, {H}, {flag})`

Class Groups & Units (bnf, bnr)

Class field theory data $a_1, \{a_2\}$ is usually bnr (ray class field), bnr, H (congruence subgroup) or bnr, χ (character on `bnr.clgp`). Any of these define a unique abelian extension of K .

units / S -units `bnfunits(bnf, {S})`
 remove GRH assumption from bnf `bnfcertify(bnf)`
 expo. of ideal x on class gp `bnfisprincipal(bnf, x, {flag})`
 expo. of ideal x on ray class gp `bnrisprincipal(bnr, x, {flag})`
 expo. of x on fund. units `bnfisunit(bnf, x)`
 ... on S -units, U is `bnfunits(bnf, S) bnfisunit(bnfs, x, U)
 signs of real embeddings of $bnf.fu$ bnfsignunit(bnf)
 narrow class group bnfnarrow(bnf)`

Class Field Theory

ray class number for modulus m `bnrclassno(bnf, m)`
 discriminant of class field `bnrdisc(a1, {a2})`
 ray class numbers, l list of moduli `bnrclassnolist(bnf, l)`
 discriminants of class fields `bnrdisclist(bnf, l, {arch}, {flag})`
 decode output from `bnrdisclist` `bnfdecodemodule(nf, fa)`
 is modulus the conductor? `bnrisconductor(a1, {a2})`
 is class field (bnr, H) Galois over K^G `bnrisgalois(bnr, G, H)`
 action of automorphism on `bnr.gen` `bnrgaloismatrix(bnr, aut)`
 apply `bnrgaloismatrix` M to H `bnrgaloisapply(bnr, M, H)`
 characters on `bnr.clgp` s.t. $\chi(g_i) = e(v_i)$ `bnrchar(bnr, g, {v})`
 conductor of character χ `bnrconductor(bnr, chi)`
 conductor of extension `bnrconductor(a1, {a2}, {flag})`
 conductor of extension $K[Y]/(g)$ `rmfconductor(bnf, g)`
 canonical projection $\text{Cl}_F \rightarrow \text{Cl}_f, f | F$ `bnrmap`
 Artin group of extension $K[Y]/(g)$ `rnfnormgroup(bnr, g)`
 subgroups of bnr , index $\leq b$ `subgrouplist(bnr, b, {flag})`
 class field defined by $H < \text{Cl}_f$ `bnrclassfield(bnr, H)`
 ... low level equivalent, prime degree `rnfkummer(bnr, H)`
 same, using Stark units (real field) `bnrstark(bnr, sub, {flag})`
 is a an n -th power in K_v ? `nfislocalpower(nf, v, a, n)`
 cyclic L/K satisf. local conditions `nfgrunwaldwang(nf, P, D, pl)`

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Logarithmic class group

logarithmic ℓ -class group	<code>bnflog(bnf, \ell)</code>
$[\tilde{e}(F_v/Q_p), \tilde{f}(F_v/Q_p)]$	<code>bnflogcoef(bnf, pr)</code>
$\exp \deg_F(A)$	<code>bnflogdegree(bnf, A, \ell)</code>
is ℓ -extension L/K locally cyclotomic	<code>rnfislocalcyclo(rnf)</code>

Ideals: elements, primes, or matrix of generators in HNF

is id an ideal in nf ?	<code>rnfideal(nf, id)</code>
is x principal in bnf ?	<code>rnfisprincipal(bnf, x)</code>
give $[a, b]$, s.t. $a\mathbf{Z}_K + b\mathbf{Z}_K = x$	<code>idealtwoelt(nf, x, \{a\})</code>
put ideal a ($a\mathbf{Z}_K + b\mathbf{Z}_K$) in HNF form	<code>idealhnf(nf, a, \{b\})</code>
norm of ideal x	<code>idealnrm(nf, x)</code>
minimum of ideal x (direction v)	<code>idealmin(nf, x, v)</code>
LLL-reduce the ideal x (direction v)	<code>idealred(nf, x, \{v\})</code>

Ideal Operations

add ideals x and y	<code>idealadd(nf, x, y)</code>
multiply ideals x and y	<code>idealmul(nf, x, y, \{flag\})</code>
intersection of ideal x with Q	<code>idealdown(nf, x)</code>
intersection of ideals x and y	<code>idealintersect(nf, x, y, \{flag\})</code>
n -th power of ideal x	<code>idealpwn(nf, x, n, \{flag\})</code>
inverse of ideal x	<code>idealinv(nf, x)</code>
divide ideal x by y	<code>idealdiv(nf, x, y, \{flag\})</code>
Find $(a, b) \in x \times y$, $a + b = 1$	<code>idealaddtoone(nf, x, \{y\})</code>
coprime integral A, B such that $x = A/B$	<code>idealnumden(nf, x)</code>

Primes and Multiplicative Structure

check whether x is a maximal ideal	<code>idealismaximal(nf, x)</code>
factor ideal x in \mathbf{Z}_K	<code>idealfactor(nf, x)</code>
expand ideal factorization in K	<code>idealfactorback(nf, f, \{e\})</code>
is ideal A an n -th power ?	<code>idealispower(nf, A, n)</code>
expand elt factorization in K	<code>nffactorback(nf, f, \{e\})</code>
decomposition of prime p in \mathbf{Z}_K	<code>idealprimedec(nf, p)</code>
valuation of x at prime ideal pr	<code>idealval(nf, x, pr)</code>
weak approximation theorem in nf	<code>idealchinese(nf, x, y)</code>
$a \in K$, s.t. $v_p(a) = v_p(x)$ if $v_p(x) \neq 0$	<code>idealappr(nf, x)</code>
$a \in K$ such that $(a \cdot x, y) = 1$	<code>idealcoprime(nf, x, y)</code>
give $bid = \text{structure of } (\mathbf{Z}_K/id)^*$	<code>idealstar(nf, id, \{flag\})</code>
structure of $(1 + \mathfrak{p})/(1 + \mathfrak{p}^k)$	<code>idealprincipalunits(nf, pr, k)</code>
discrete log of x in $(\mathbf{Z}_K/bid)^*$	<code>ideallog(nf, x, bid)</code>
<code>idealstar</code> of all ideals of norm $\leq b$	<code>ideallist(nf, b, \{flag\})</code>
add Archimedean places	<code>ideallistarch(nf, b, \{ar\}, \{flag\})</code>
init <code>modpr</code> structure	<code>nfmodprinit(nf, pr, \{v\})</code>
project t to \mathbf{Z}_K/pr	<code>nfmodpr(nf, t, modpr)</code>
lift from \mathbf{Z}_K/pr	<code>nfmodprlift(nf, t, modpr)</code>

Galois theory over \mathbf{Q}

conjugates of a root θ of nf	<code>nfgaloisconj(nf, \{flag\})</code>
apply Galois automorphism s to x	<code>nfgaloisapply(nf, s, x)</code>
Galois group of field $\mathbf{Q}[x]/(f)$	<code>polgalois(f)</code>
initializes a Galois group structure G	<code>galoisinit(pol, \{den\})</code>
character table of G	<code>galoischartable(G)</code>
conjugacy classes of G	<code>galoisconjclasses(G)</code>
$\det(1 - \rho(g)T)$, χ character of ρ	<code>galoischarpoly(G, \chi, \{o\})</code>
$\det(\rho(g))$, χ character of ρ	<code>galoischarpoly(G, \chi, \{o\})</code>
action of p in <code>nfgaloisconj</code> form	<code>galoispermopol(G, \{p\})</code>
identify as abstract group	<code>galoisidentify(G)</code>
export a group for GAP/MAGMA	<code>galoisexport(G, \{flag\})</code>
subgroups of the Galois group G	<code>galoissubgroups(G)</code>
is subgroup H normal?	<code>galoisnormal(G, H)</code>

subfields from subgroups	<code>galoisubfields(G, \{flag\}, \{v\})</code>
fixed field	<code>galoisfixedfield(G, perm, \{flag\}, \{v\})</code>
Frobenius at maximal ideal P	<code>idealfrobenius(nf, G, P)</code>
ramification groups at P	<code>idealarmgroups(nf, G, P)</code>
is G abelian?	<code>galoisisabelian(G, \{flag\})</code>
abelian number fields/ \mathbf{Q}	<code>galoisubcyclo(N, H, \{flag\}, \{v\})</code>

The galpol package

query the package: polynomial	<code>galoisgetpol(a, b, \{s\})</code>
... : permutation group	<code>galoisgetgroup(a, b)</code>
... : group description	<code>galoisgetname(a, b)</code>

Relative Number Fields (rnf)

Extension L/K is defined by $T \in K[x]$.

absolute equation of L	<code>rnfequation(nf, T, \{flag\})</code>
is L/K abelian?	<code>rnfisabelian(nf, T)</code>
relative <code>nfalgtobasis</code>	<code>rnfalgtobasis(rnf, x)</code>
relative <code>nfbasistoalg</code>	<code>rnfbasistoalg(rnf, x)</code>
relative <code>idealhnf</code>	<code>rnfidealhnf(rnf, x)</code>
relative <code>idealmul</code>	<code>rnfidealmul(rnf, x, y)</code>
relative <code>idealtwoelt</code>	<code>rnfidealtwoelt(rnf, x)</code>

Lifts and Push-downs

absolute \rightarrow relative representation for x	<code>rnfeltabstorel(nf, x)</code>
relative \rightarrow absolute representation for x	<code>rnfeltreltoabs(nf, x)</code>
lift x to the relative field	<code>rnfeltup(rnf, x)</code>
push x down to the base field	<code>rnfeltdown(rnf, x)</code>
idem for x ideal: <code>(rnfideal)reltoabs, abstorel, up, down</code>	

Norms and Trace

relative norm of element $x \in L$	<code>rnfeltnorm(rnf, x)</code>
relative trace of element $x \in L$	<code>rnfelttrace(rnf, x)</code>
absolute norm of ideal x	<code>rnfidealnrmabs(rnf, x)</code>
relative norm of ideal x	<code>rnfidealnrmrel(rnf, x)</code>
solutions of $N_{K/\mathbf{Q}}(y) = x \in \mathbf{Z}$	<code>bnfisintnorm(bnf, x)</code>
is $x \in \mathbf{Q}$ a norm from K ?	<code>bnfisnorm(bnf, x, \{flag\})</code>
initialize T for norm eq. solver	<code>rnfisnorminit(K, pol, \{flag\})</code>
is $a \in K$ a norm from L ?	<code>rnfisnorm(T, a, \{flag\})</code>
initialize t for Thue equation solver	<code>thueinit(f)</code>
solve Thue equation $f(x, y) = a$	<code>thue(t, a, \{sol\})</code>
characteristic poly. of $a \bmod T$	<code>rnfcharpoly(nf, T, a, \{v\})</code>

Factorization

factor ideal x in L	<code>rnfidealfactor(rnf, x)</code>
$[S, T]: T_{i,j} S_i$; S primes of K above p	<code>rnfidealprimedec(rnf, p)</code>

Maximal order \mathbf{Z}_L as a \mathbf{Z}_K -module

relative <code>polredbest</code>	<code>rnfpolredbest(nf, T)</code>
relative <code>polredabs</code>	<code>rnfpolredabs(nf, T)</code>
relative Dedekind criterion, prime pr	<code>rnfddedekind(nf, T, pr)</code>
discriminant of relative extension	<code>rnfdisc(nf, T)</code>
pseudo-basis of \mathbf{Z}_L	<code>rnfpsudobasis(nf, T)</code>

General \mathbf{Z}_K -modules: $M = [\text{matrix, vec. of ideals}] \subset L$

relative HNF / SNF	<code>nfnhf(nf, M), nfnfnf</code>
multiple of $\det M$	<code>nfdetint(nf, M)</code>
HNF of M where $d = \text{nfdetint}(M)$	<code>nfnhfmod(x, d)</code>
reduced basis for M	<code>rnflllgram(nf, T, M)</code>
determinant of pseudo-matrix M	<code>rnfdet(nf, M)</code>
Steinitz class of M	<code>rnfsteinitz(nf, M)</code>

\mathbf{Z}_K -basis of M if \mathbf{Z}_K -free, or 0	<code>rnfhnfbasis(bnf, M)</code>
n -basis of M , or $(n + 1)$ -generating set	<code>rnfbasis(bnf, M)</code>
is M a free \mathbf{Z}_K -module?	<code>rnfisfree(bnf, M)</code>

Associative Algebras

A is a general associative algebra given by a multiplication table mt (over \mathbf{Q} or \mathbf{F}_p); represented by al from `algtableinit`.

create al from mt (over \mathbf{F}_p)	<code>algtableinit(mt, \{p = 0\})</code>
group algebra $\mathbf{Q}[G]$ (or $\mathbf{F}_p[G]$)	<code>alggroup(G, \{p = 0\})</code>
center of group algebra	<code>alggroupcenter(G, \{p = 0\})</code>

Properties

is (mt, p) OK for <code>algtableinit</code> ?	<code>algisassociative(mt, \{p = 0\})</code>
multiplication table mt	<code>algmtable(al)</code>
dimension of A over prime subfield	<code>algdim(al)</code>
characteristic of A	<code>algchar(al)</code>
is A commutative?	<code>algiscommutative(al)</code>
is A simple?	<code>algissimple(al)</code>
is A semi-simple?	<code>algissemisimple(al)</code>
center of A	<code>algcenter(al)</code>
Jacobson radical of A	<code>algradical(al)</code>
radical J and simple factors of A/J	<code>algsimpledec(al)</code>

Operations on algebras

create A/I , I two-sided ideal	<code>algquotient(al, I)</code>
create $A_1 \otimes A_2$	<code>algtensor(al1, al2)</code>
create subalgebra from basis B	<code>algsubalg(al, B)</code>
quotients by ortho. central idempotents e	<code>algcentproj(al, e)</code>
isomorphic alg. with integral mult. table	<code>algmakeintegral(mt)</code>
prime subalgebra of semi-simple A over \mathbf{F}_p	<code>algprimesubalg(al)</code>
find isomorphism $A \cong M_d(\mathbf{F}_q)$	<code>algsplit(al)</code>

Operations on lattices in algebras

lattice generated by cols. of M	<code>alglathnf(al, M)</code>
... by the products xy , $x \in lat1$, $y \in lat2$	<code>alglatmul(al, lat1, lat2)</code>
sum $lat1 + lat2$ of the lattices	<code>alglatadd(al, lat1, lat2)</code>
intersection $lat1 \cap lat2$	<code>alglatinter(al, lat1, lat2)</code>
test $lat1 \subset lat2$	<code>alglatsubset(al, lat1, lat2)</code>
generalized index $(lat2 : lat1)$	<code>alglatindex(al, lat1, lat2)</code>
$\{x \in al \mid x \cdot lat1 \subset lat2\}$	<code>alglatlefttransporter(al, lat1, lat2)</code>
$\{x \in al \mid lat1 \cdot x \subset lat2\}$	<code>alglatrighttransporter(al, lat1, lat2)</code>
test $x \in lat$ (set $c = \text{coord. of } x$)	<code>alglatcontains(al, lat, x, \{\&c\})</code>
element of lat with coordinates c	<code>alglatelement(al, lat, c)</code>

Operations on elements

$a + b, a - b, -a$	<code>algadd(al, a, b), algsub, algneg</code>
$a \times b, a^2$	<code>algmul(al, a, b), algsqrt</code>
a^n, a^{-1}	<code>algpow(al, a, n), alginv</code>
is x invertible ? (then set $z = x^{-1}$)	<code>algisinv(al, x, \{\&z\})</code>
find z such that $x \times z = y$	<code>algdivl(al, x, y)</code>
find z such that $z \times x = y$	<code>algdivr(al, x, y)</code>
does s s.t. $x \times z = y$ exist? (set it)	<code>algisdivl(al, x, y, \{\&z\})</code>
matrix of $v \mapsto x \cdot v$	<code>algtomatrix(al, x)</code>
absolute norm	<code>algnorm(al, x)</code>
absolute trace	<code>algtrace(al, x)</code>
absolute char. polynomial	<code>algcharpoly(al, x)</code>
given $a \in A$ and polynomial T , return $T(a)$	<code>algpoleval(al, T, a)</code>
random element in a box	<code>algrandom(al, b)</code>

Based on an earlier version by Joseph H. Silverman

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Central Simple Algebras

A is a central simple algebra over a number field K ; represented by al from **alginit**; K is given by a nf structure.

create CSA from data	alginit ($B, C, \{v\}, \{maxord = 1\}$)
multiplication table over K	$B = K, C = mt$
cyclic algebra $(L/K, \sigma, b)$	$B = rnf, C = [\sigma, b]$
quaternion algebra $(a, b)_K$	$B = K, C = [a, b]$
matrix algebra $M_d(K)$	$B = K, C = d$
local Hasse invariants over K	$B = K, C = [d, [PR, HF], HI]$

Properties

type of al (mt, CSA)	algtype (al)
dimension of A over \mathbf{Q}	algdim ($al, 1$)
dimension of al over its center K	algdim (al)
degree of A ($= \sqrt{\dim_K A}$)	algdegree (al)
al a cyclic algebra $(L/K, \sigma, b)$; return σ	algaut (al)
... return b	algb (al)
... return L/K , as an rnf	algsplittingfield (al)
split A over an extension of K	algsplittingdata (al)
splitting field of A as an rnf over center	algsplittingfield (al)
multiplication table over center	algremltable (al)
places of K at which A ramifies	algramifiedplaces (al)
Hasse invariants at finite places of K	alghassef (al)
Hasse invariants at infinite places of K	alghassei (al)
Hasse invariant at place v	alghasse (al, v)
index of A over K (at place v)	algindex ($al, \{v\}$)
is al a division algebra? (at place v)	algisdivision ($al, \{v\}$)
is A ramified? (at place v)	algisramified ($al, \{v\}$)
is A split? (at place v)	algissplit ($al, \{v\}$)

Operations on elements

reduced norm	algnorm (al, x)
reduced trace	algtrace (al, x)
reduced char. polynomial	algcharpoly (al, x)
express x on integral basis	algalgtobasis (al, x)
convert x to algebraic form	algbasistoalg (al, x)
map $x \in A$ to $M_d(L)$, L split. field	algtomatrix (al, x)

Orders

Z -basis of order \mathcal{O}_0	algbasis (al)
discriminant of order \mathcal{O}_0	algdisc (al)
Z -basis of natural order in terms \mathcal{O}_0 's basis	alginvbasis (al)

Based on an earlier version by Joseph H. Silverman

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