The functions in this section depend on \([Q(f) : Q(\chi)]\) as above. Initialize symbol \(f\) attached to \(f\) evaluate symbol \(s\) on path \(p\) Peterson product of \(f\) and \(g\) period polynomial of \(f\) period polynomials for eigenbasis \(FS\) modulo \(C\)

\[\begin{align*}
\text{forms embedded into } C \\
\text{Given a modular form } f \text{ in } M_k(\Gamma_0(N), \chi) \text{ its field of definition } Q(f) \\
\text{has } n = [Q(f) : Q(\chi)] \text{ embeddings into the complex numbers. If } n = 1, \\
\text{the following functions return a single answer, attached to} \\
\text{the canonical embedding of } f \text{ in } C[\overline{\chi}]; \text{ else a vector of } n \text{ results,} \\
\text{corresponding to the } n \text{ conjugates of } f. \\
\text{complex embeddings of } Q(f) \text{ and } \text{evalf}(f, r) \\
\text{L-function attached to } f \\
\text{...eigenforms of new space } M \\
\text{Periods and symbols}
\end{align*}\]
Modular Symbols

Let $G = \Gamma_0(N)$, $V_k = \mathbb{Q}[X,Y][k^{-2}]$, $L_k = \mathbb{Z}[X,Y][k^{-2}]$. We let $\Delta = \text{Div}_{X_0(N),V_k}$; an element of $\Delta$ is a path between cusps of $X_0(N)$ via the identification $[b] - [a]$ to the path from $a$ to $b$. A path is coded by the pair $[a,b]$, where $a,b$ are rationals or $\infty$, denoting the point at infinity $(1:0)$.

Let $M_k(G) = \text{Hom}_\mathbb{Z}(\Delta, V_k) \cong H^1_{\text{cusp}}(X_0(N),V_k)$; an element of $M_k(G)$ is a $V_k$-valued modular symbol. There is a natural decomposition $M_k(G) = M_k(G)^+ \oplus M_k(G)^-$ under the action of the $*$ involution, induced by complex conjugation. The $\text{msinit}$ function computes either $M_k$ ($\epsilon = 0$) or its $\pm$-parts ($\epsilon = \pm 1$) and fixes a minimal set of $\mathbb{Z}[G]$-generators $(g_i)$ of $\Delta$.

- Initialize $M = M_k(\Gamma_0(N))^\epsilon$.
- The level $M$.
- The weight $k$.
- The sign $\epsilon$.
- Farey symbol attached to $G$.

$\mathbb{Z}[G]$-generators $(g_i)$ and relations for $\Delta$.

Create a symbol

Eisenstein symbol attached to cusp $c$.

Cuspidal symbol attached to $E/Q$.

Symbol having given Hecke eigenvalues is a symbol?

Operations on symbols

- The list of all $s(g_i)$.
- Evaluate symbol $s$ on path $p = [a,b]$.

Operators on subspaces

An operator is given by a matrix of a fixed $\mathbb{Q}$-basis. $H$, if given, is a stable $\mathbb{Q}$-subspace of $M_k(G)$: operator is restricted to $H$.

- Matrix of Hecke operator $T_p$ or $U_p$.
- Matrix of Atkin-Lehner $w_Q$.
- Matrix of the $*$ involution.

Subspaces

A subspace is given by a structure allowing quick projection and restriction of linear operators. Its fist component is a matrix with integer coefficients whose columns for a $\mathbb{Q}$-basis. If $H$ is a Hecke-stable subspace of $M_k(G)^+$ or $M_k(G)^-$, it can be split into a direct sum of Hecke-simple subspaces. To a simple subspace corresponds a single normalized newform $\sum_n a_n q^n$.

Eisenstein subspace $E_k(G)^\epsilon$.

New part of $S_k(G)^\epsilon$.

Split $H$ into simple subspaces (of dim $\leq d$).

Dimension of a subspace.

For attached newform $\text{msdelta}(M, H, \{B\})$.

$\mathbb{Z}$-structure from $H^1(G, L_k)$ on subspace $A$.

Overconvergent symbols and $p$-adic $L$ functions

Let $M$ be a full modular symbol space given by $\text{msinit}$ and $p$ be a prime. To a classical modular symbol $\phi$ of level $N$ ($v_p(N) \leq 1$), which is an eigenvector for $T_p$ with non-zero eigenvalue $a_p$, we can attach a $p$-adic $L$-function $L_p$. The function $L_p$ is defined on continuous characters of $\text{Gal}(\mathbb{Q}(\mu_{p^\infty})/\mathbb{Q})$; in GP we allow characters $(\chi_n)_{n \geq 1}$, where $(s_1,s_2)$ are integers, $r$ is the Teichmüller character and $\chi$ is the cyclotomic character.

The symbol $\phi$ can be lifted to an overconvergent symbol $\Phi$, taking values in spaces of $p$-adic distributions (represented in GP by a list of moments modulo $p^k$).

$\text{mspadicinit}$ precomputes data used to lift symbols. If $\text{flag}$, it speeds up the computation by assuming that $v_p(a_p) = 0$ if $\text{flag} = 0$ (fastest), and that $v_p(a_p) \geq \text{flag}$ otherwise (faster as $\text{flag}$ increases).

$\text{mspadicmoments}$ computes distributions $\mu$ attached to $\Phi$ allowing to compute $L_p$ to high accuracy.

Based on an earlier version by Joseph H. Silverman

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